Intermediate Algebra

Functions and Graphs

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Lesson 1 Linear Models

Activity 1 Making and Reading a Graph

In 2000, the average sea level, L, was 24 millimeters above normal, and it has been rising at a rate of 3 mm per year.

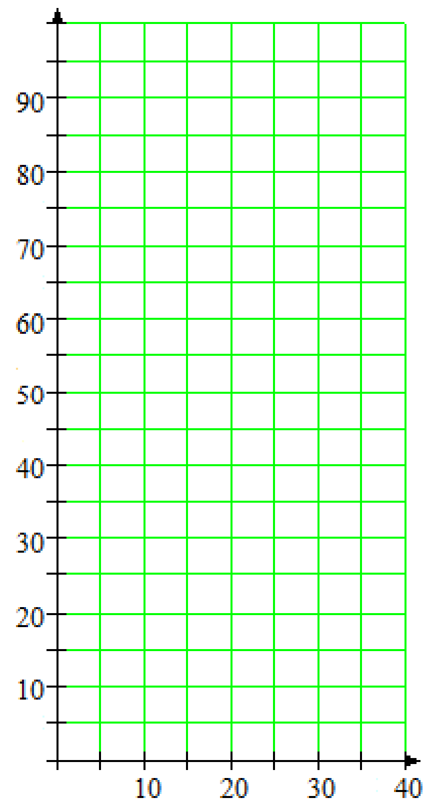
a. Complete the table of values for the sea level t years after 2000.

t (years)	0	5	10	15	20
L (mm)					

b. Write an equation for the sea level, L, in terms of t, the number of years since 2000.

$L =$

c. Graph the equation on the grid. Label the axes with the correct variables.



To answer questions (d) and (e), read the graph. Show your work on the graph:

d. According to this model, what will the sea level be in 2020?

e. How long will it be before the sea level is 6 centimeters above average? (How many millimeters in 1 centimeter?)

f. Now use algebra to find the answers for parts (d) and (e) above. Compare your answers to the values you read from the graph.

(d): (e):

g. Another global warming concern is rising temperature. Since 1950, the global mean temperature in degrees Celsius can be modeled by the equation

$$T = 13.9 + 0.013\,t$$

where t is the number of years since 1950. What do the constants in this equation tell us about global temperature?

13.9 tells us:

0.013 tells us:

Activity 2 A Decreasing Graph

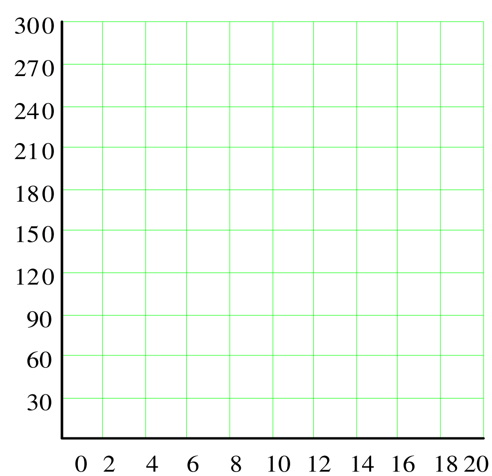
Silver Lake has been polluted by industrial waste products. In 2000 the concentration of toxic chemicals in the water was 285 **p**arts **p**er **m**illion (ppm). Local environmental officials started a program to reduce the concentration by 15 ppm each year.

a. Complete the table of values showing the concentration, C, of toxic chemicals t years after 2000. Then write your answers as ordered pairs.

Year	t	C
2000	0	
2005	5	
2010	10	
2015	15	

(t, C)
(0,)
(5,)
(10,)
(15,)

b. Label the axes, plot the ordered pairs on the grid, and connect them with a straight line.



c. In one or two complete sentences, explain why you should stop the graph at the horizontal axis.

d. Interpret the point $(16, 45)$ for this situation. (Use complete sentences!)

e. Use the graph to find those years when the concentration will be below 120 ppm but above 30 ppm. Show your work on the graph.

f. Write an equation for the concentration, C, of toxic chemicals t years after 2000.

g. Write and solve an inequality to verify your answer to part (e).

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Analyze linear models given by a description in words, by a graph, by a table of values, or by an equation.
- Write an equation for a linear model described in words, in the form:
  $y =$ (starting value) $+$ (rate) $\times\, t$
- Interpret an ordered pair of values for a linear model.
- Make a graph from a table of values by choosing appropriate scales for the axes and plotting points.
- Evaluate an algebraic expression.
- Solve linear equations and inequalities.
- Use a graph to evaluate an expression or solve an equation.

Check Your Understanding

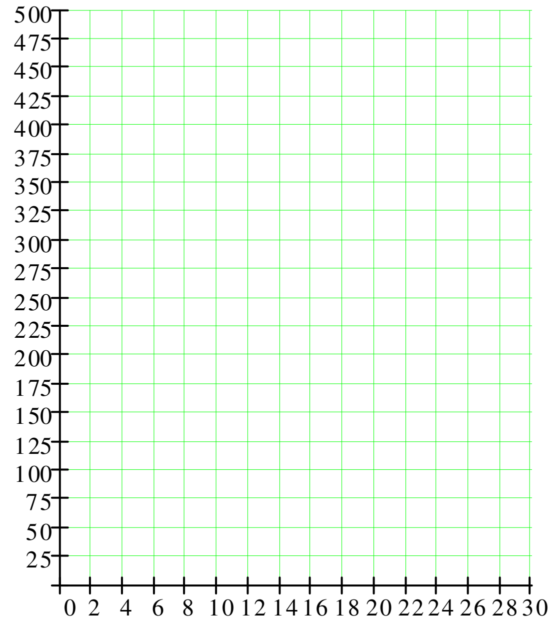
1. In Activity 1b, you wrote an equation for sea level, L. What was the starting value, and what was the rate?
2. In part (d) of Activity 1, did you evaluate an expression or solve an equation?
3. In part (e) of Activity 1, you solved an equation by using the graph. Did you start by locating the given value on the horizontal axis, or on the vertical axis?
4. In Activity 2, which variable goes on the horizontal axis?
5. In part (e) of Activity 2, you solved an inequality. On which axis did your answers appear?

Lesson 2 Intercepts

Activity 1 Interpreting the Intercepts

Copper is the third most widely used metal, behind iron and aluminum. In 2007 the US Geological Survey estimated that there are 490 million tons of usable copper reserves left in the world. The global consumption of copper in 2007 was 17.8 million tons.

a. Supposing that we continue to consume copper at the same annual rate, write an equation for the amount of copper, C, in millions of tons, remaining t years after 2007.



b. Complete the table of values.

Year	2010	2015	2020
t			
C			

c. Graph the equation.

d. Use algebra to find the coordinates of each intercept of the graph.

t	C
0	
	0

e. Explain what the intercepts tell us about the problem.

The C-intercept is __________ . It tells us:

The t-intercept is __________ . It tells us:

Activity 2 General Form for a Linear Equation

The owner of a gas station has $19,200 to spend on unleaded gas this month. Regular unleaded costs him $2.40 per gallon, and premium unleaded costs $3.20 per gallon.

a. Write algebraic expressions to answer the questions:

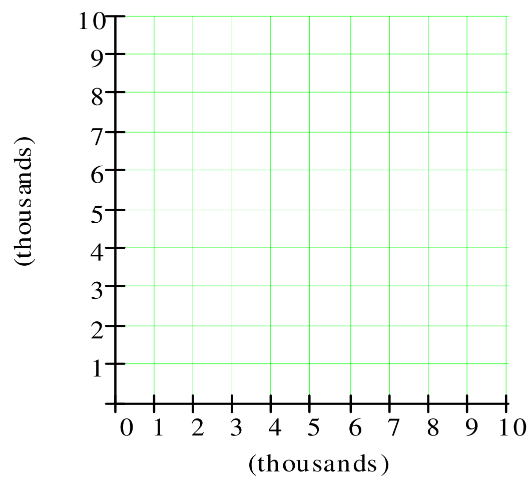
How much do x gallons of regular cost?

How much do y gallons of premium cost?

b. Write an equation that relates the amount of regular unleaded gasoline, x, the owner can buy and the amount of premium unleaded, y.

c. Find the intercepts and use the intercept method to sketch the graph.

x	y
0	
	0



d. Explain what the intercepts tell us about the amount of gasoline.

The x-intercept is ______ . It tells us:

The y-intercept is ______ . It tells us:

e. If the gas station owner would like to buy 5000 gallons of regular gasoline, how many gallons of premium can he buy? Label this point on the graph.

f. What is the meaning of the point $(2000, 4500)$ on the graph?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Interpret the intercepts of the graph of a linear model.
- Write a model in general linear form, $Ax + By = C$.
- Find the intercepts of the graph of a linear equation.
- Graph a line by the intercept method.
- Solve a linear equation for y in terms of x.

Check Your Understanding

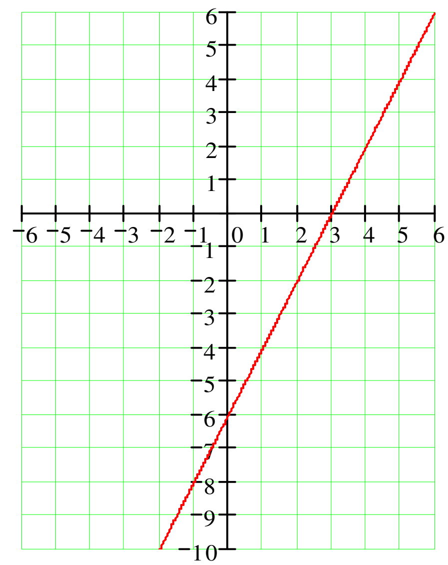
1. In Activity 1, how did you use algebra to find the intercepts of the graph?
2. In Activity 1, which intercept gives the starting value of C?
3. The equation you wrote in Activity 2 was different in form from the equation you wrote in Activity 1. How?
4. In Activity 2, what did the variables x and y represent?

Lesson 3 Graphs and Equations

Activity 1 Evaluating and Solving

Use the graph of $y = 2x - 6$ to:

y = 2x - 6



a. Evaluate $y = 2x - 6$ for $x = -1$:
Circle the point on the graph with $x = -1$.
What is its y-coordinate?

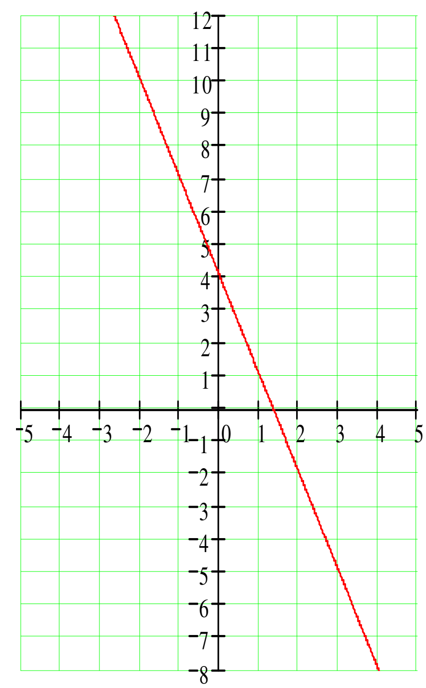
b. Solve $y = 2x - 6$ when $y = 4$:
Circle the point on the graph with $y = 4$.
What is its x-coordinate?

c. Find the answers to parts (a) and (b) algebraically, and compare with the answers you obtained from the graph.

Activity 2 Solving Equations Graphically

Use the graph to solve the equations.
(Show your work by making a dot at the correct point on the graph for each equation.)

y = 4 - 3x



a. Solve $4 - 3x = 10$
The y-value is 10.
Find the x-value.

Solution:

b. Solve $-2 = 4 - 3x$
The y-value is -2.
Find the x-value.

Solution:

c. Solve $0 = 4 - 3x$

The y-value is 0.
Find the x-value.

Solution:

d. Look back at your work in Activity 2. Use the graph to show that the following three statements are different ways of saying the same thing:

1. The point $(-1, 7)$	lies on the graph of	$y = 4 - 3x$
2. $x = -1,\ y = 7$	is a solution of the equation	$y = 4 - 3x$
3. $x = -1$	is a solution of the equation	$7 = 4 - 3x$

Activity 3 Solving Inequalities Graphically

a. Solve $2x + 1 < 7$

Circle the point **on the graph** with $y = 7$, and label its coordinates.

Circle the x-value of this point on the x-axis. This is the **boundary value**, and it separates the x-axis into two parts.

Choose one x-value to the left of the boundary point, and one value to the right. (For example, $x = 0$ and $x = 4$.) Mark these **test values** on the x-axis.

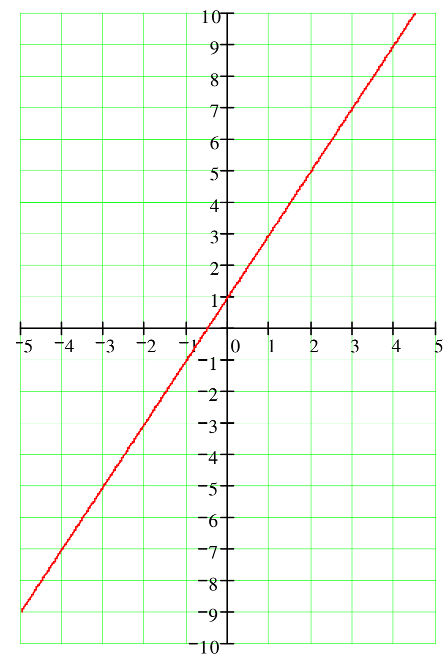
Circle the points **on the graph** corresponding to each test value. Label each point with its coordinates.

Which test value has its corresponding $y < 7$? Shade the portion of the x-axis that includes that test value.

Solution:

Choose any shaded x-value and verify that it is a solution.

y = 2X + 1



b. Solve $2x + 1 \geq -5$

Circle the point **on the graph** with $y = -5$, and label its coordinates.

Circle the x-value of this point on the x-axis. This is the **boundary value**, and it separates the x-axis into two parts.

Choose one x-value to the left of the boundary point, and one value to the right. Mark these **test values** on the x-axis.

Circle the points **on the graph** corresponding to each test value. Label each point with its coordinates.

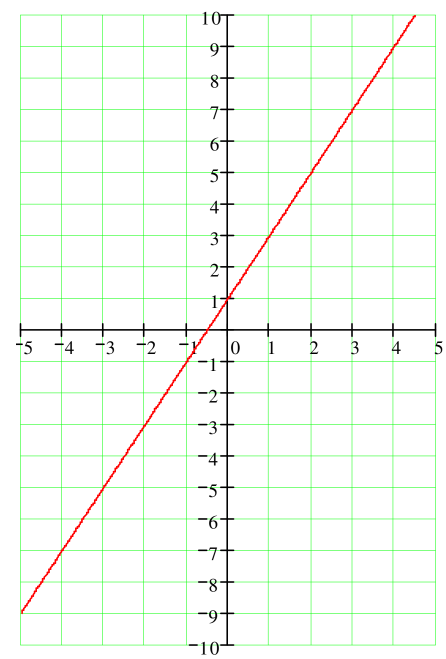
Which test value has its corresponding $y \geq -5$?

Shade the portion of the x-axis that includes that test value.

Solution:

Choose any shaded x-value and verify that it is a solution.

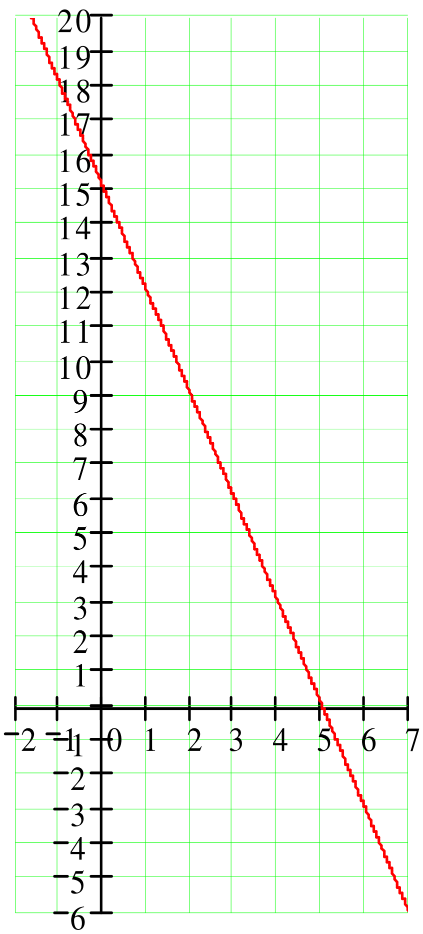
y = 2X + 1



Steps for solving an inequality graphically:
1. Locate the boundary value on the x-axis.
2. Find the y-values at a test value on either side of the boundary value and check them in the inequality.
3. Shade the correct portion of the x-axis.

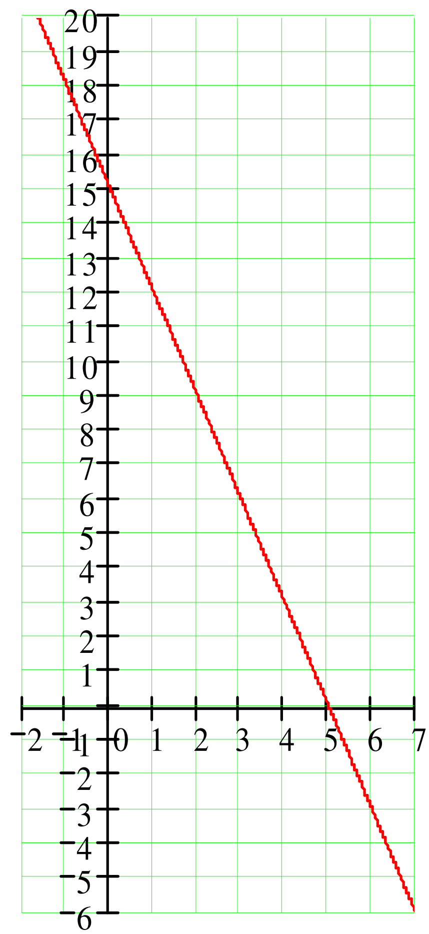
c. Use the graphs of $y = 15 - 3x$ to solve the inequalities:

Solve $15 - 3x \geq 9$



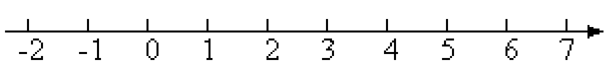
Solution:

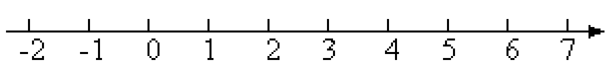
Solve $6 > 15 - 3x$



Solution:

Graph each solution on a number line:





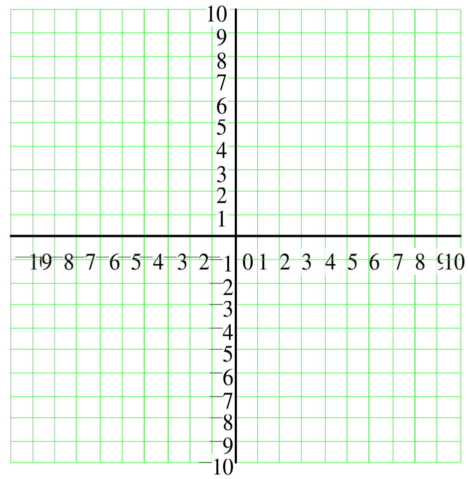
How does the graph on the number line compare to your solution on the x-axis in each problem?

Activity 4 Windows and Tracing

1. **a.** Graph $7 - 2y = 4x$ in the **standard window**. (First solve for y.)

$y =$

b. Copy your graph onto the grid.



2. **a.** Graph $y = -18 + \frac{6}{5}x$ in the standard window. Describe what you see.

b. Change the window settings to

Xmin $= -20$ Xmax $= 20$
Ymin $= -20$ Ymax $= 20$

Press $\boxed{\textbf{TRACE}}$ and use the left and right arrow keys. What do the numbers at the bottom of the screen tell you?

c. Return the **Trace** bug to the y-intercept. Press $\boxed{\textbf{ZOOM}}$ **8** $\boxed{\textbf{ENTER}}$ to access the **ZInteger** window. Now use the **Trace** feature again. What is different this time?

d. Press $\boxed{\textbf{WINDOW}}$. What window settings does **ZInteger** give you?

Activity 5 Solving Equations and Inequalities

1. **a.** Graph the equation $y = 32x - 42$ in the window

Xmin $= -4.7$ Xmax $= 4.7$ Xscl $= 1$
Ymin $= -250$ Ymax $= 50$ Yscl $= 25$

b. Sketch a picture of your calculator's screen in the space at right. Label the ends of the axes with the minimum and maximum values.

c. Use the **Trace** feature to solve the equation $32x - 42 = -122$

d. Verify your answer algebraically by substituting your x-value into the equation.

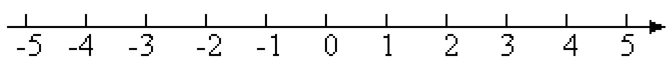
2. **a.** Graph the equation $y = 1.3x + 2.4$
Set the **WINDOW** to

Xmin $= -4.6$ Xmax $= 4.8$
Ymin $= -10$ Ymax $= 10$

b. Sketch a picture of your calculator's screen in the space at right. Label the ends of the axes with the minimum and maximum values.

c. Use your graph to solve the inequality $1.3x + 2.4 < 8.51$. Show your work on your sketch of the graph.

d. Graph the solutions on a number line.



e. Verify your answer by solving the inequality algebraically.

Wrap-Up

In this Lesson, we worked on the following skills related to linear models:

- Recognize a solution of an equation or an inequality
- Solve a linear inequality, and graph the solutions on a number line
- Recognize a solution of an equation in two variables
- Use the connection between the graph of an equation and its solutions
- Use a graphing calculator to graph an equation
- Use a graph to solve an equation or inequality in one variable

Check Your Understanding

1. What are the two skills compared in Activity 1? What is the difference in the methods for performing the two skills?

2. What was different about the problems in Activity 3, compared to those in Activity 2?

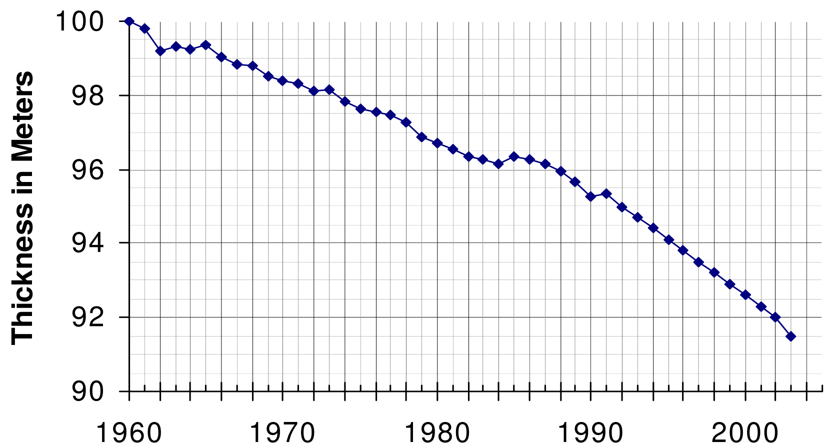
3. How were the solutions you found in Activity 3 different from the solutions in Activity 2?

4. In Activity 3, how do you decide which side of the boundary point to shade?

Lesson 4 Slope

Activity 1 Calculating Rate of Change

The graph shows how the thickness of a typical land-based glacier has changed over 43 years.



a. What was the **total change**, ΔH, in thickness from 1960 to 2003?

Year, t	Thickness, H
1960	
2003	

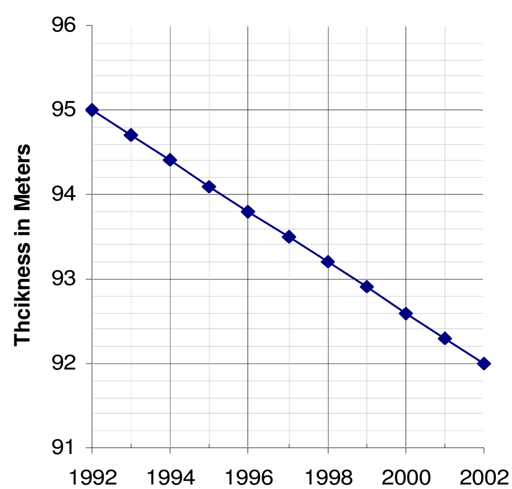
$\Delta H =$

Calculate the **average yearly change** in thickness, $\dfrac{\Delta H}{\Delta t}$, over that time interval. Give units with your answers.

$$\frac{\Delta H}{\Delta t} =$$

b. The graph appears to be almost linear from 1992 to 2002. Read the graph to complete the table.

Year, t	Thickness, H
1992	
2002	



c. Calculate the slope of the graph from 1992 to 2002. Include units in your answer.

d. What does the slope tell us about glaciers?

Activity 2 Slope and Linear Models

The taxi fare in three different cities is described below. In each city, you pay an initial charge when you get into the taxi, and then your fare is based on the distance you travel. Each city uses a different distance unit to compute the fare.

City	Initial Charge	Distance Unit	Charge per Unit
Boston	1.45	$\frac{1}{8}$ mile	0.30
Honolulu	2.25	$\frac{1}{4}$ mile	0.75
New York	2.50	$\frac{1}{5}$ mile	0.40

a. Compute the charge per mile in each city. (Do not include the initial charge.) In which city do taxis charge the highest mileage rate?

b. Write a linear model for the taxi fare in each city, using miles as the input variable. (**Hint:** What is the initial value for each model?)

c. In which city do taxis charge the lowest fare for a 5 mile ride?

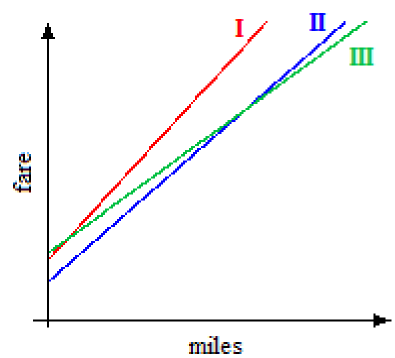
d. For what distance are the taxi fares in Boston and New York equal? (**Hint**: Use the appropriate models from part (b).)

e. Choose the correct graph for each city. Explain how you decided.

Boston:

Honolulu:

New York:



Activity 3 Interpreting Slope as a Rate

The table shows the amount of salt that can be dissolved in a beaker of water at different temperatures.

Temperature (°C), T	10	12	15	21	25	40	52
Salt (g), S	33	34	35.5	38.5	40.5	48	54

a. Which variable is the input, and which is the output?

input: output:

If you plot the data, which variable goes on the horizontal axis, and which on the vertical?

horizontal: vertical:

b. Will the points lie on a straight line? Why or why not?

c. Calculate the slope, and interpret it as a rate of change. Include units in your answer.

d. Use the slope to answer the question: If you increase the temperature 5°C from the current temperature, how much more salt will dissolve?

Wrap-Up

In this Lesson, we worked on the following skills and goals:

- Use ratios and rates for comparison.
- Compute and interpret a rate of change.
- Interpret slope as a rate of change.
- Illustrate slope on a graph.
- Use the definition of slope to compute Δy or Δx.
- Recognize a linear relationship by its constant rate of change.
- Write a linear model using slope and initial value

Check Your Understanding

1. What mathematical concept did you study in Activity 1?
2. What two numbers did you need to write each linear model in part (b) of Activity 2?
3. What algebraic technique did you use in part (d) of Activity 2?
4. Describe one new skill or idea you learned from Activity 3.

Lesson 5 Equations of Lines

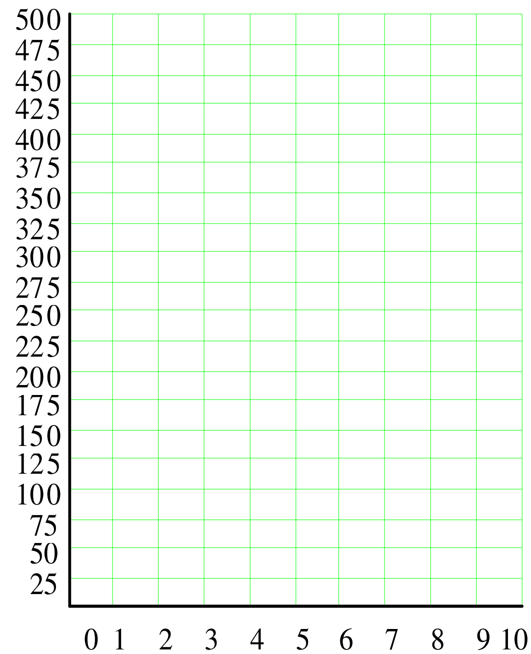
Activity 1 Slope-Intercept Form

In England, oven cooking temperatures are often given as Gas Marks rather than degrees Fahrenheit. The table shows the equivalent oven temperatures for various Gas Marks.

Gas Mark	3	5	7	9
Degrees (F)	325	375	425	475

a. Plot the data and draw a line through the data points.

b. Calculate the slope of your line.



Estimate the y-intercept from the graph.

c. Write an equation that gives the temperature, T, in degrees Fahrenheit, in terms of the Gas Mark, G.

Activity 2 Working with Formulas

a. We now have several new formulas to work with. Solve each formula for the indicated variable:

$m = \dfrac{\Delta y}{\Delta x}$ $\qquad\Delta y =$

$\Delta y = y_2 - y_1$ $\qquad y_2 =$

$\Delta x = x_2 - x_1$ $\qquad x_2 =$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad y_2 - y_1 =$

b. Suppose you are studying the graph of a line, $y = mx + b$. Explain the difference between what the statements $x = 2$ and $\Delta x = 2$ mean.

Activity 3 Finding a Linear Model

Sea water does not freeze at exactly 32° F because of its salinity. The temperature at which water freezes depends on its dissolved mineral content. A common unit for measuring salinity is parts per thousand, or ppt. For example, salinity of 8 ppt means 8 grams of dissolved salts in each kilogram of water. Here are some data for the freezing temperature of water.

Salinity (ppt), S	8	12	20
Freezing temperature (°F), T	31.552	31.328	30.88

a. Do these data points describe a linear model? Why or why not?

b. Use the point-slope formula to find a linear equation for freezing temperature, T, in terms of salinity, S.

Step 1: Find the slope $\quad m = \dfrac{T_2 - T_1}{S_2 - S_1} =$

Step 2: Use the point-slope formula $\quad T - T_1 = m(S - S_1)$

c. What is the salinity of water that freezes at 32° F?

d. Sea water has an average salinity of 35 ppt. What is the freezing point of sea water?

e. The conversion formula from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. What is the freezing point of sea water in degrees Celsius?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Identify the slope and y-intercept from the equation for a line
- Interpret the slope and y-intercept in context
- Use the coordinate formula to calculate slope
- Use the point-slope formula to find an equation for a line
- Find a linear model from two data points

Check Your Understanding

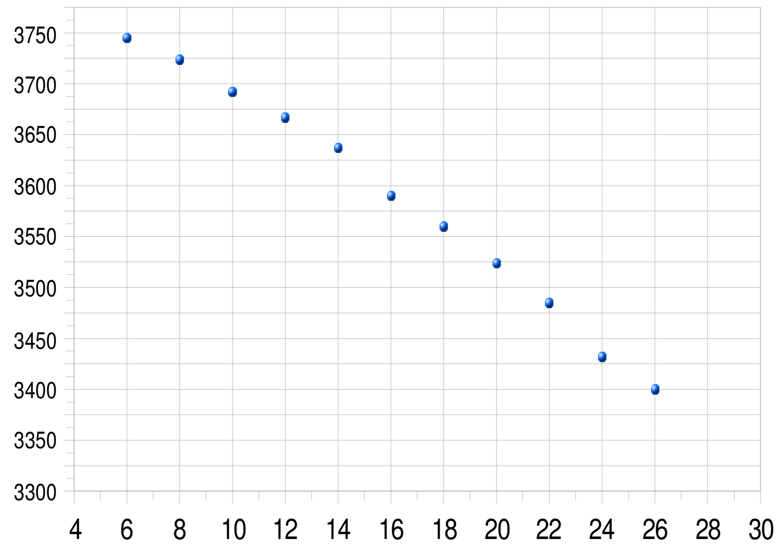
1. Which formula did you use to find the equation of the line in Activity 1?
2. If you graph the data in Activity 3, which variable goes on the vertical axis?
3. When you calculate a slope using data, how do you know which variable goes in the numerator of the slope ratio?
4. Which two formulas do you need to find the equation of a line that goes through two given points?

Lesson 6 Linear Regression

Activity 1 Line of Best Fit

The scatterplot shows the area, A, of the Amazon rain forest remaining, in thousands of square kilometers, t years after 1980.

t	A
6	3745
8	3724
10	3692
12	3667
14	3637
16	3590
18	3560
20	3524
22	3485
24	3432
26	3400



a. Use a straightedge to draw a line of best fit for the data.

b. Choose two points on the regression line. (Do not choose data points; choose points on your line.)

Points:

c. Use the point-slope formula to find the equation of your regression line.

d. Use your regression equation to predict when the rain forest will be completely destroyed.

e. Comment on your results: What factors might affect your prediction?

Activity 2 Least-Squares Regression

a. Use the data for the Amazon rain forest and follow the instructions below to compute and graph the least-squares regression line on your calculator.

Step 1. Start by clearing out old data.

a. Press **STAT** **ENTER** to see the data lists.

b. If there are old data in the lists, press △ to select **L1**, then press **CLEAR** then **ENTER**.

c. Press ▷ and repeat step (b) to clear the other lists.

Step 2. "Edit" the data.

a. Enter your x-values in the **L1** column.

b. Press the right arrow ▷ and enter your y-values in the **L2** column.

Step 3. Make a scatterplot.

a. Press **2nd** **Y =** to get the statistics plot menu.

b. Press **1** **ENTER** to select the first statistics plot.

c. After **Type**, select the scatterplot (it's the first picture).

d. Enter **L1** for the **Xlist**, and **L2** for the **Ylist** (**L1** is **2nd** **1**, **L2** is **2nd** **2**).

e. Press **ZOOM** **9** to see the scatterplot.

Step 4. Compute the regression coefficients.

a. Press **STAT** ▷ to get the calculation menu.

b. Press **4** for linear regression (**LinReg ($ax+b$)**), then **ENTER**. (If you want to use lists other than **L1** and **L2**, enter them after **LinReg ($ax+b$),** separated by **,** .)

c. Jot down the regression equation on a piece of paper. (Round the coefficients to three decimal places.)

Step 5. Graph the regression line.

a. Press **Y =** and clear out any old equations.

b. Press **VARS** **5** ▷ ▷ **1** to copy the regression equation after **Y =**. OR: Just enter the regression equation by hand, copying from your piece of paper.

c. Press **GRAPH**.

b. How does the least-squares regression line compare to the line of best fit you calculated in Activity 1?

c. Do any of the data points lie on the least-squares regression line?

Don't forget to turn off your statistics plots after you finish!

a. Press $\boxed{\textbf{2nd}}$ $\boxed{\textbf{Y} =}$ to get the statistics plot menu.
b. Press $\boxed{\textbf{4}}$ $\boxed{\textbf{ENTER}}$ to turn off all the plots.

Activity 3 Interpolation and Extrapolation

It has been proposed that unemployment is higher in communities where a large percentage of workers own their own homes. The data show the percent of owner-occupied housing and the unemployment rate in several European nations. We will calculate a regression line to model the data.

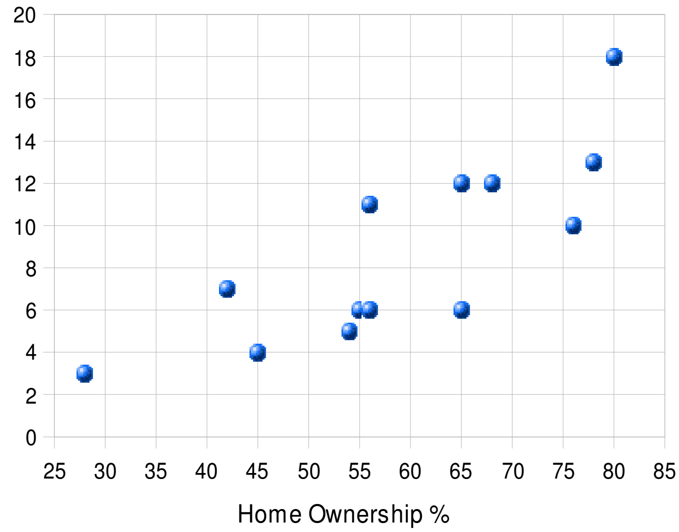
a. Use a straightedge to draw a line of best fit for the scatterplot shown on the next page.

b. Use a calculator to find the equation of the least-squares regression line. Round the coefficients to two decimal places.

c. Evaluate the least-squares formula for 40% home ownership and for 70% home ownership.
(Note: Do *not* change the percents to decimals!)

Country	Home Owners. %	Unemp. Rate %
Austria	54	5
Belgium	65	12
Denmark	55	6
Finland	78	13
France	56	11
Great Britain	65	6
Ireland	76	10
Italy	68	12
Netherlands	45	4
Spain	80	18
Sweden	56	6
Switzerland	28	3
West Germany	42	7

d. Use the values from part (c) to graph the least-squares regression line on the scatterplot, and compare to your line of best fit.



e. What is the slope of the least-squares regression line? What is its meaning for this situation?

f. What is the vertical intercept of the least-squares regression line? What is its meaning for this situation?

g. What unemployment rate does the model predict for a society with 100% home ownership?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Sketch a line of best fit for a scatterplot
- Find an equation for a line of best fit
- Make estimates using interpolation and extrapolation
- Use a calculator for least-squares regression

Check Your Understanding

1. Should you adjust a line of best fit so that it passes through two of the data points? Why or why not?

2. In Activity 1, why don't we use data points to find the equation of the line of best fit?

3. In Activity 3, how did your line of best fit compare to the least-squares regression line?

4. In which part(s) of Activity 3 did you use interpolation, and in which part(s) did you use extrapolation?

Lesson 7 Linear Systems

Activity 1 Solving a System by Graphing

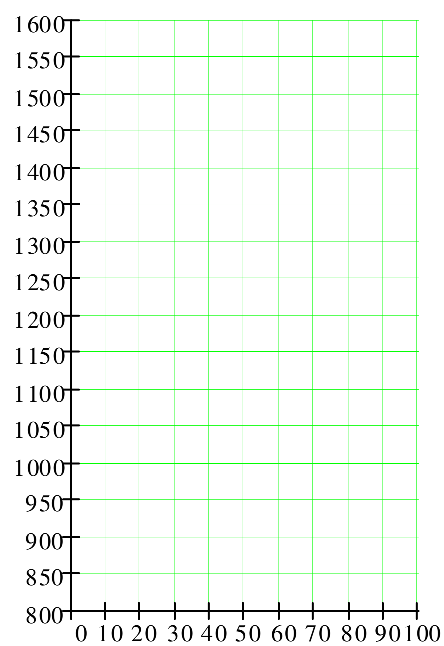
Delbert and Francine are buying appliances for their new home. They have a choice of two different refrigerators: a standard model that sells for \$1000 and an energy-efficient model at a price of \$1200. The standard model costs \$6 per month to run, and the energy-efficient model costs \$2 per month.

a. Write an equation for S, the total cost of purchasing and running the standard model for t months.

b. Write an equation for E, the total cost of purchasing and running the energy-efficient model for t months.

c. Complete the table of values and graph both equations on the grid.

t	20	40	60
S			
E			



d. Read your graph: What is the intersection point of the two graphs? What do the coordinates of the point tell us?

e. Use algebra to find the answer to part (d).

Activity 2 Using a Graphing Calculator

Francine wants to combine oats and wheat to make a cereal with 10 grams of protein per cup. Oat flakes provide 11 grams of protein per cup. Wheat flakes provide 8.5 grams of protein per cup. How many cups of each grain will she need for a box of cereal containing 3 cups?

a. Describe the two unknown quantities:

x :

y :

b. Complete the table.

	Cups	Grams of protein per cup	Grams of protein
Oats			
Wheat			
Mixture			

c. Write two equations, one about the amount of each grain in the mixture, and one about the amount of protein in the mixture.

d. Solve each equation for y, and graph them on your calculator. (Hint: What are the largest and smallest possible values for x and y? Use these values to choose a window.)

e. Use the **intersect** command to find the intersection point. What do the coordinates of the intersection point tell us?

Activity 3 How Many Solutions?

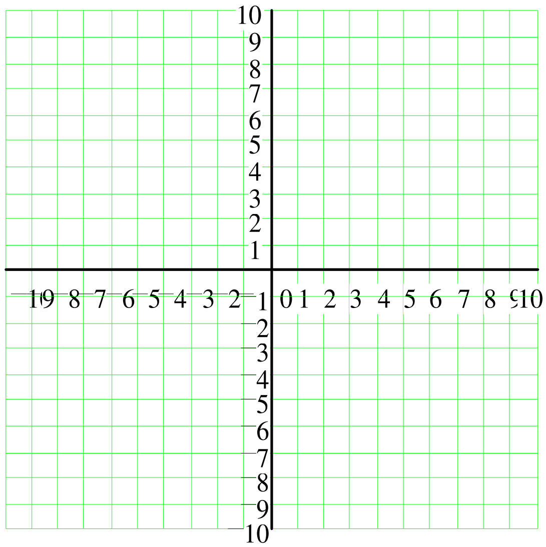
a. Graph the system on the grid.

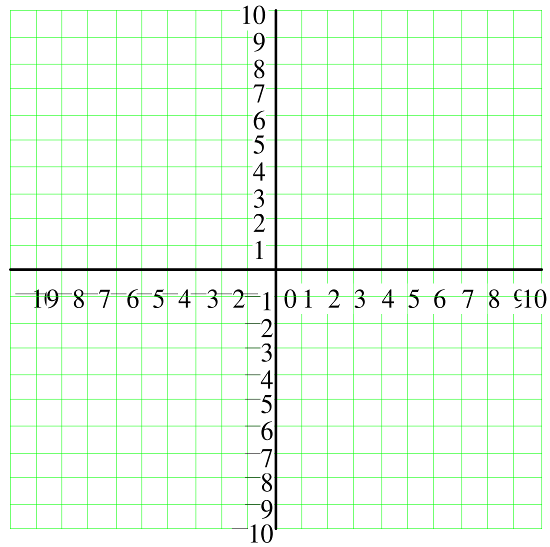
I

$$3x - 2y = 9$$
$$x = \frac{2}{3}y + 3$$

II

$$y = -3x + 6$$
$$6x + 2y = 15$$





b. How many solutions are there?

System **I** ____________________ System **II** ________________________

c. Write each equation in slope-intercept form. What do you notice?

d. Without graphing the equations, how can you tell how many solutions a system has?

e. There are three possible forms for a system of two linear equations in two variables. Refer to the Reading to find the names of these three forms, and how many solutions each form has.

f. Is it possible for a system of two linear equations in two variables to have exactly two solutions? Why or why not?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Verify a solution of a system,
- Solve a linear system graphically.
- Identify inconsistent and dependent systems.
- Model applications in two variables.

Check Your Understanding

1. In Activity 1, how did you use a graph to solve the system?
2. How did you solve the system in Activity 1 algebraically?
3. In Activity 2, each equation corresponds to one of the columns of the table. Which column goes with each equation?
4. Delbert says that the solution of the system $y = 2x,\ y = 12 - x$ is $x = 4$. What is wrong with his answer?

Lesson 8 Algebraic Solution of Systems

Activity 1 Interest Problems

Carmella has \$1200 invested in two stocks; one earns 8% per year and the other earns 12% per year. The income from the 8% stock is \$3 more than the income from the 12% stock. How much did she invest in each stock?

a. Choose variables for the two unknown quantities, and fill in the table.

x :

y :

	Principal	Interest Rate	Interest
First stock			
Second stock			
Total		--------	--------

b. Write one equation about the amount Carmella **invested**.

c. Write a second equation about Carmella's annual **interest**.

d. Solve the system and answer the question in the problem.

Activity 2 Mixture Problems

Amal plans to make 10 liters of a 17% acid solution by mixing a 20% acid solution with a 15% acid solution. How much of each should she use?

a. Choose variables for the unknown quantities, and fill in the table.

x :

y :

	Liters	% Acid	Amount of Acid
First Solution			
Second Solution			
Mixture			

b. Write one equation about the amount of **solution** Amal needs.

c. Write a second equation about the amount of **acid** in the solutions.

d. Solve the system and answer the question in the problem.

Activity 3 Motion Problems

Because of prevailing winds a flight from Detroit to Denver, a distance of 1120 miles, takes 4 hours on Econoflite, while the return trip takes 3.5 hours. What were the speed of the airplane and the speed of the wind?

a. Choose variables for the unknown quantities, and fill in the table.

x :

y :

	Rate	Time	Distance
Detroit to Denver			
Denver to Detroit			

b. Write one equation about the trip from Detroit to Denver.

c. Write a second equation about the return trip.

d. Solve the system and answer the question in the problem.

Sample Templates for Common Applications of Linear Systems

1. Interest. Typical interest problems follow this pattern:

	Principal	Interest rate	Interest
First investment			
Second investment			
Total			

a. Fill in the first two columns with information from the problem.
b. Apply the formula $\boldsymbol{Pr = I}$ (Principal $\times$ rate $=$ Interest) to fill in the third column.
c. Write two equations, one about the principal (first column), and one about the interest (third column)

2. Mixture. Typical mixture problems follow this pattern:

	Amount of Solution	Percent of Important Ingredient	Amount of Important Ingredient
First solution			
Second solution			
Mixture			

a. Fill in the first two columns with information from the problem.
b. Apply the formula $\boldsymbol{Wr = P}$ (Whole $\times$ percentage rate $=$ Part) to fill in the third column.
c. Write two equations, one about the entire solution (first column), and one about the important ingredient (third column)

The strength or percentage of the mixture is called the **weighted average** of the strengths of the two ingredients, and the "weights" are the amounts of each ingredient. If the strengths of the two ingredients are p_1 and p_2, and the amounts of each are w_1 and w_2, then

$$p_1 w_1 + p_2 w_2 = p_m w_m$$

where p_m and w_m are the strength and amount of the mixture, and $w_1 + w_2 = w_m$.

3. Motion. Typical motion problems follow this pattern:

	Rate	Time	Distance
First trip			
Second trip			

a. Fill in the first two columns with information from the problem.
b. Apply the formula $\boldsymbol{RT = D}$ (Rate $\times$ Time $=$ Distance) to fill in the third column.
c. Write two equations about the problem: Use the columns to make sure you are relating rates to rates, times to times, or distances to distances.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Solve systems using substitution.
- Solve systems using elimination.
- Identify dependent and inconsistent systems.
- Model and solve applied problems with a system of equations.

Check Your Understanding

1. For the interest problem in Activity 1, we wrote one equation about ____________ and a second equation about ____________ .

2. For the mixture problem in Activity 2, we wrote one equation about ____________ and a second equation about ____________ .

3. If you are traveling upstream in a river, how does the current affect your speed?

4. Which of the three types of problems, interest, mixture, or motion, is the easiest?

Lesson 9 Gaussian Reduction

Activity 1 Triangular Systems

Use **back-substitution** to solve the system.

$$\begin{aligned} 3x + y - 2z &= 10 \\ 2y + 3z &= 4 \\ 6z &= -24 \end{aligned}$$

Solution:

Activity 2 Gaussian Elimination

a. Follow the steps to solve the system.

(1) $x - 2y + 4z = -3$
(2) $3x + y - 2z = 12$
(3) $2x + y - 3z = 11$

There are no fractions in the equations, so we start with Step 2.

Step 2 Eliminate x from Equations (1) and (2):

Call your new equation (A):

Step 3 Eliminate x from Equations (1) and (3):

Call your new equation (B):

Step 4 Eliminate y from Equations (A) and (B):
(*Hint:* Divide Equation (A) by 7 first.)

Step 5 Form a triangular system:

Solve the triangular system by back-substitution.

Solution:

b. Solve the system on a separate sheet of paper. Begin by clearing the fractions from each equation.

$$\begin{aligned} x + 2y + \frac{1}{2}z &= 0 \\ x + \frac{3}{5}y - \frac{2}{5}z &= \frac{1}{5} \\ 4x - 7y - 7z &= 6 \end{aligned}$$

Activity 3 Making the Smart Choice

Sometimes a wise choice of steps can shorten the process.

(1)	$3y + z = 3$	Notice that there is no x-term in Equation (1).
(2)	$-2x + 3y = 7$	Equation (1) becomes Equation (A): Step 2 is done!
(3)	$3x + 2z = -6$	What is the smart choice for Step 3?

Now finish the solution.

Activity 4 Application

A manufacturer of office supplies makes three types of file cabinet: two-drawer, four-drawer, and horizontal. The manufacturing process is divided into three phases: assembly, painting, and finishing. A two-drawer cabinet requires 3 hours to assemble, 1 hour to paint, and 1 hour to finish. The four-drawer model takes 5 hours to assemble, 90 minutes to paint, and 2 hours to finish. The horizontal cabinet takes 4 hours to assemble, 1 hour to paint, and 3 hours to finish. The manufacturer employs enough workers for 500 hours of assembly time, 150 hours of painting, and 230 hours of finishing per week. How many of each type of file cabinet should he make in order to use all the hours available?

Step 1 Represent the number of each model of file cabinet by a different variable.

Number of two-drawer cabinets: x
Number of four-drawer cabinets: y
Number of horizontal cabinets: z

Step 2 Organize the information into a table. (Convert all times to hours.)

	2-Drawer	4-Drawer	Horizontal	Total Available
Assembly				
Painting				
Finishing				

Write three equations describing the time constraints in each of the three manufacturing phases. For example, the assembly phase requires $3x$ hours for the two-drawer cabinets, $5y$ hours for the four-drawer cabinets, and $4z$ hours for the horizontal cabinets, and the sum of these times should be the time available, 500 hours.

(Assembly time) (1)
(Painting time) (2)
(Finishing time) (3)

Step 3 Solve the system. Follow the steps suggested below.

1. Clear the fractions from the second equation.

2. Subtract Equation (1) from 3 times Equation (3) to obtain a new Equation (4).

3. Subtract Equation (2) from twice Equation (3) to obtain a new Equation (5).

4. Equation (4) and (5) form a 2×2 system in y and z. Subtract Equation (5) from Equation (4) to obtain a new Equation (6).

5. Form a triangular system with equations (3), (4), and (6). Use back-substitution to complete the solution.

Step 4 You should find the following solution: The manufacturer should make 60 two-drawer cabinets, 40 four-drawer cabinets, and 30 horizontal cabinets.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to linear models:

- Use back-substitution to solve a triangular system
- Use Gaussian elimination to solve a 3×3 linear system
- Model a problem with three linear equations in three unknowns

Check Your Understanding

1. What is a triangular system?
2. In Gaussian elimination, after you eliminate one of the variables from a pair of equations, what should you do next?
3. What should you do if one of the original equations has only two variables?
4. What is the difference between an inconsistent system and a dependent system?

Lesson 10 Extraction of Roots

Activity 1 Volume

You are designing a new coffee maker. The coffee filter must be 8.4 centimeters tall and shaped like a cone. The volume of the filter will therefore depend on how wide the opening is (the radius of the cone).

a. Write a formula for the filter's volume V in terms of its widest radius r (at the top of the filter). (*Hint*: Formulas for volume given in the Reading.)

Simplify your formula by multiplying the constants together.

b. Complete the table of values. Round the volumes to one decimal place.

r (cm)	1	2	3	4	5	6	7	8
V (cm^3)								

c. Look at the table of values. If you double the radius of the filter, by what factor does the volume increase?

d. Write and solve an equation to answer the question: If the volume of the filter is 302.4 cubic centimeters, what is its radius?

e. Graph the volume equation on your calculator. (Use the table of values to help you choose a window.) Sketch your graph in the space at right (don't forget to scale the axes). Label the coordinates of the point on the graph that corresponds to the filter in part (**d**).

Activity 2 Compound Interest

Find the formula for **compound interest** in the Reading:

Cyril would like to invest $5000 for two years in a money market account that pays interest compounded annually.

a. Write a formula for the balance, B, of Cyril's account after two years in terms of the interest rate, r.

b. Complete the table showing Cyril's account balance after two years for various interest rates.

r	0.02	0.04	0.06	0.08
B				

c. If Cyril would like to have $6250 in two years, what interest rate must the account pay?

d. Graph the formula for Cyril's account balance on your calculator. Sketch your graph in the space at right (don't forget to scale the axes). Label the coordinates of the point on the graph that corresponds to the balance in part (**c**).

Activity 3 Extraction of Roots

1. Solve each equation by extraction of roots.

a. $(x+3)^2 = 4$

b. $(3x+1)^2 = 25$

2. Solve. Give your answers two ways:
i) exact values and
ii) approximations rounded to thousandths.

a. $6(x-3)^2 = 30$

b. $530 + 60(2+x)^2 = 1000$

3. Solve each formula for the requested variable.

a. $V = \frac{1}{3}\pi r^2 h$ for r

b. $d = h - \frac{1}{2}gt^2$ for t

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Recognize a quadratic equation and its parameters
- Graph a parabola by plotting points
- Solve a quadratic equation by extraction of roots
- Use formulas that involve quadratic terms

Check Your Understanding

1. Sketch a graph that illustrates why a quadratic equation can have two solutions.
2. In part (d) of Activity 1, what technique did you use to solve the equation?
3. Explain how you found the solution to part (c) of Activity 2, being careful to list all the steps you used.
4. Explain how to deal with the fractions in Problem 3 of Activity 3.

Lesson 11 Intercepts and Factors

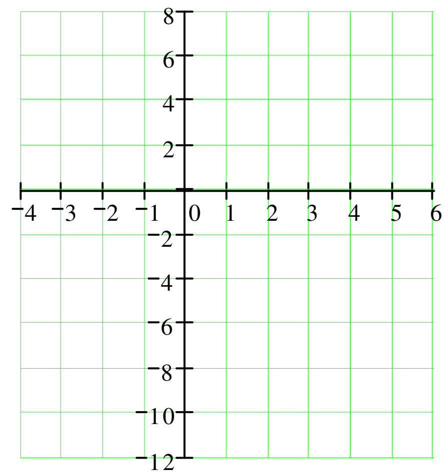
Activity 1 A Typical Quadratic Graph

a. Verify that $y = (2x - 7)(x + 1)$ is a quadratic equation. (*Hint*: Expand the right side.)

b. Graph $y = (2x - 7)(x + 1)$ in the friendly window

Xmin $= -9.4$ Xmax $= 9.4$
Ymin $= -10$ Ymax $= 10$

Sketch the graph on the grid.



c. Use the **Trace** feature to locate the x-intercepts of the graph.

d. Solve the equation

$$(2x - 7)(x + 1) = 0$$

e. Explain how your answers to parts (**c**) and part (**d**) are related.

Activity 2 X-Intercepts

1. a. Graph all three equations in the standard window, and sketch the graphs in the space at the right.
What do you notice about the x-intercepts?

(1) $y = x^2 + 2x - 15$
(2) $y = 3(x^2 + 2x - 15)$
(3) $y = 0.2(x^2 + 2x - 15)$

> **Multiplying a quadratic expression by a constant does not change the x-intercepts of the graph.**

b. Solve by factoring: $20x^2 - 40x - 700 = 0$

2. a. Graph $y = 0.1(x^2 + 9x - 360)$ on your calculator, using the **ZInteger** setting.
b. Locate the x-intercepts of the graph.

c. Use the x-intercepts to write the quadratic expression in factored form. (Do not try to factor the expression!)

Activity 3 Perimeter and Area

Do all rectangles with the same perimeter, say 36 inches, have the same area?

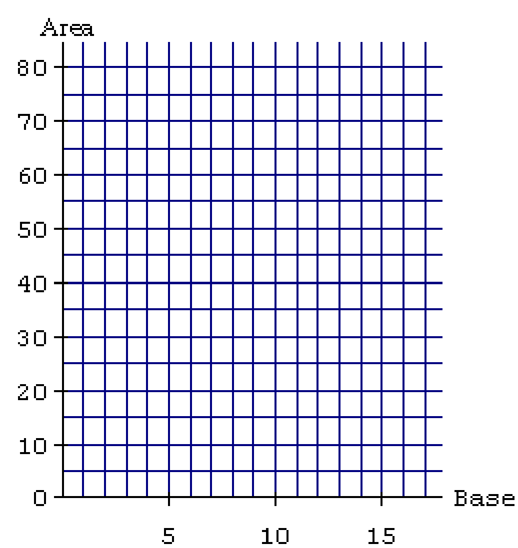
1. a. Sketch a rectangle with perimeter 36 inches and base 10 inches, and label its dimensions. (To find the height of the rectangle, reason as follows: The base plus the height makes up half of the rectangle's perimeter.) What is its area?

b. Sketch a rectangle with perimeter 36 inches and base 12 inches, and label its dimensions. What is its area?

Base	Height	Area
10	8	80
12	6	72
3		
14		
5		
17		
9		
2		
11		
4		
16		
15		
1		
6		
8		
13		
7		

2. The table shows the bases of various rectangles with perimeter 36 inches. Fill in the height and the area of each.

3. Plot the points with coordinates (Base, Area). (We will not use the heights of the rectangles.) Connect your data points with a smooth curve.



4. What is the largest area you found among rectangles with perimeter 36 inches? What is the base for that rectangle? What is its height?

Area:

Base:

Height:

5. Let x represent the base of the rectangle. Write an expression for the height of the rectangle in terms of x. (Hint: If the perimeter of the rectangle is 36 inches, what is the sum of the base and the height?)

Height:

Write an expression for the area of the rectangle in terms of x.

Area:

Activity 4 Modeling

A rancher has 360 yards of fence to enclose a rectangular pasture. If she uses a riverbank to border one side of the pasture, she can enclose 16,000 square yards of land. What will the dimensions of the pasture be then?

We'll solve this problem in three different ways. First, make a sketch of the pasture:

1. Using a Table of Values

Notice that you only have 360 yards of fence. So if you decide how wide the pasture is going to be, the length of the pasture is determined. See the table for two examples.

a. Use the perimeter of the pasture, 360 yards, to write a formula for the length of the pasture in terms of its width. (Be careful: remember that one side of the pasture does not need any fence!)

Length $=$

b. Make a table that shows the areas of pastures of various widths.

c. Continue the table until you find the pasture whose area is 16,000 square yards.

Dimensions of pasture:

Width	Length	Area
10	340	3400
20	320	6400

2. Using a Graph

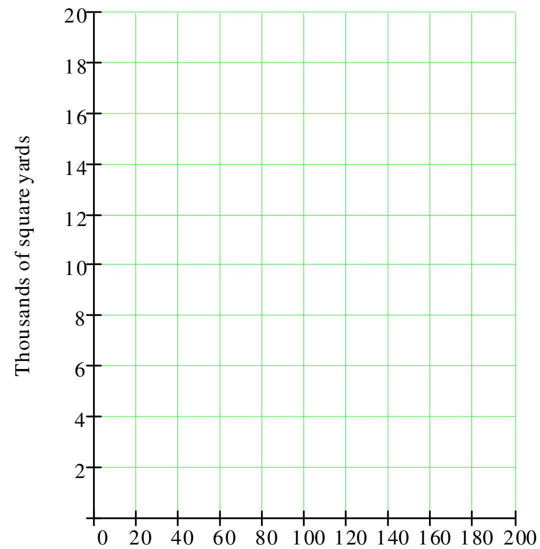
On your sketch of the pasture in part (1), label the width of the pasture as x.

a. Write an expression for the length of the pasture if its width is x. (Hint: how did you compute the length of the pasture in part (1)?

Length $=$

b. Write an expression for the area A of the pasture if its width is x.

Area $=$



c. Graph the equation for A on your calculator, using the window

Xmin $= 0$ Xmax $= 188$
Ymin $= 0$ Ymax $= 17000$

Then copy the graph onto the grid.

d. Use the **TRACE** feature to find the pasture of area 16,000 square yards. Label the correct point on your graph.

Dimensions of pasture:

3. Using an Equation

a. Refer to steps (a) and (b) of part 2, and write an equation for the area A of the pasture in terms of its width x.

Area $=$

b. Substitute $A = 16,000$ and solve your equation algebraically.

Dimensions of pasture:

Which of the three methods do you prefer? Why?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Solve quadratic equations by factoring
- Find a quadratic equation with given solutions
- Find the x-intercepts of a parabola
- Solve problems involving perimeter and area or the Pythagorean theorem

Check Your Understanding

1. Delbert says that the solutions of the equation $4(x-2)(x+1)=0$ are $x=4$, $x=2$, and $x=-1$. Do you agree? Why or why not?

2. If the perimeter of a rectangle is 56 inches, and its width is w inches, write an expression for its length.

3. How can you use a graph to factor a quadratic expression?

4. In part 2 of Activity 4, what did the two variables on the graph represent? What happened to the second variable as you increased the first variable?

Lesson 12 Graphing Parabolas

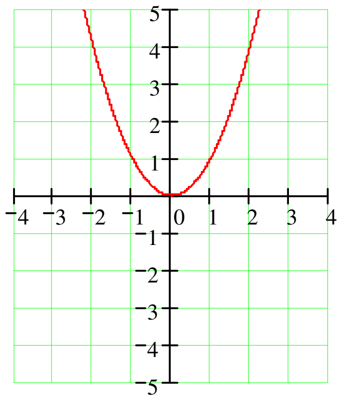
Activity 1 Graphing Practice

1. Complete the table of values, then sketch the graph and label the coordinates of three points on the graph.

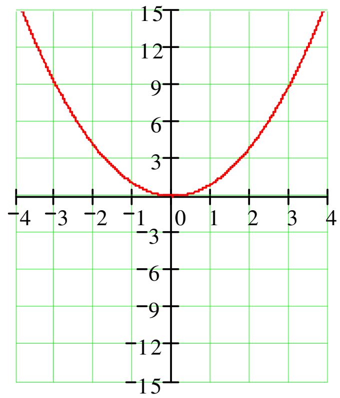
x	-2	-1	0	1	2
y					

x	-2	-1	0	1	2
y					

a. $y = \frac{1}{2}x^2$



b. $y = -3x^2$



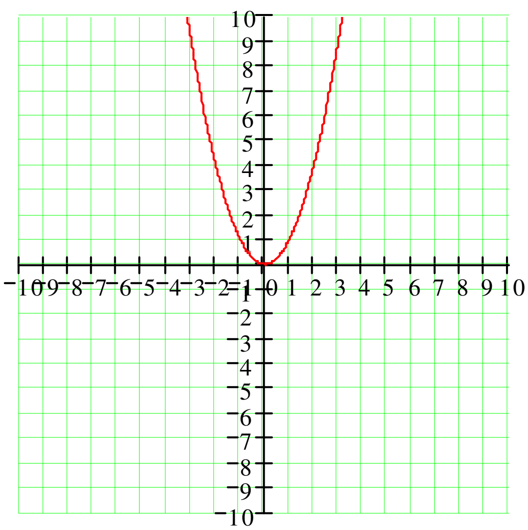
c. Compare each graph to the basic parabola, $y = x^2$. How is it different?

2. Complete the table of values, then sketch the graph and label the coordinates of three points on the graph.

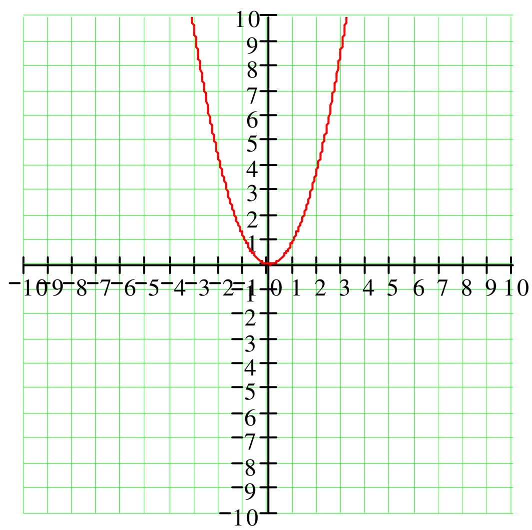
x	-4	-3	-1	0	1	3	4
y							

x	-4	-3	-1	0	1	3	4
y							

a. $y = x^2 - 9$



b. $y = 9 - x^2$



1) Find the x-intercepts of each graph.

2) Find the vertex of each graph.

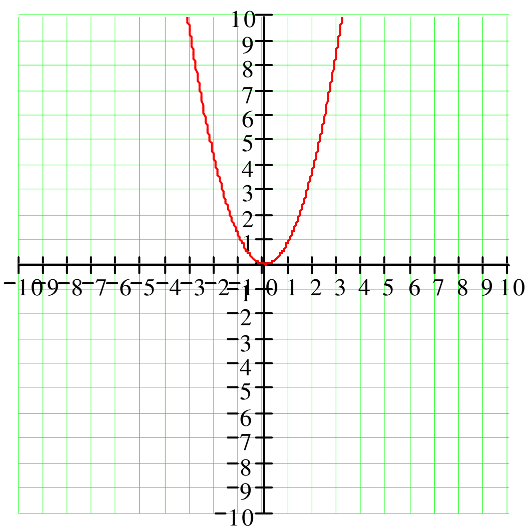
c. How do these graphs compare to the basic parabola, $y = x^2$?

3. Complete the table of values, then sketch the graph and label the coordinates of three points on the graph.

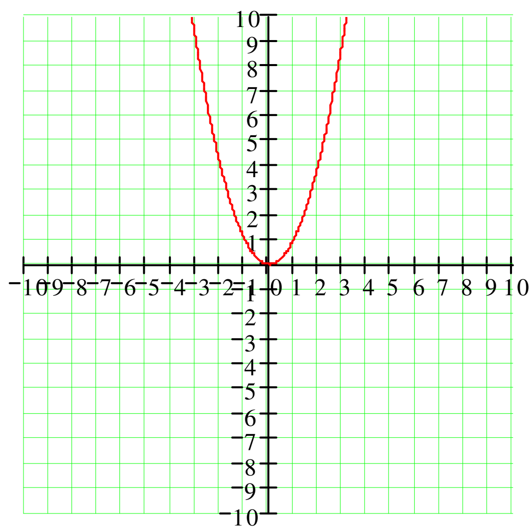
x	-6	-4	-3	-2	0	1
y						

x	-1	0	1	$\frac{3}{2}$	2	3	4
y							

a. $y = x^2 + 6x$



b. $y = 2x^2 - 6x$



1) Find the x-intercepts of each graph.

2) Find the vertex of each graph.

c. How do these graphs compare to the basic parabola, $y = x^2$?

Activity 2 Intercepts and Vertex

1. How do we find the y-intercept of a parabola from its equation?

2. How do we find the x-intercepts of a parabola from its equation?

3.

Question: How do we find the average of two numbers? Answer: Add them up, and divide the result by 2. Or, in other words, take half of their sum.

Write an expression for the average of the numbers P and Q.

4. a. If the x-intercepts of a parabola are $x = p$ and $x = q$, what is the x-intercept of its vertex?

b. If the x-intercepts of a parabola are $x = 0$ and $x = \frac{-b}{a}$, what is the x-intercept of its vertex?

c. How do we find the y-coordinate of the vertex from the equation?

Activity 3 Summary

1. Write a description of a typical parabola, using the words vertex, axis of symmetry, y-intercept, and x-intercept. Illustrate your description with figures.

2. Describe how each of the three parameters a, b, and c affect the graph of the basic parabola:

a. $y = ax^2$

b. $y = x^2 + c$

c. $y = x^2 + bx$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Describe the effect of each of the parameters $a, b,$ and c on the graph of a parabola
- Find the x- and y-intercepts of a parabola
- Find the vertex of a parabola
- Sketch the graph of a parabola

Check Your Understanding

1. What are the x-intercepts of the graph of $y = x^2 - 16$? What are the x-intercepts of the graph of $y = x^2 - 16x$?
2. What is the axis of symmetry of the graph of $y = x^2 - 16$? What is the axis of symmetry of the graph of $y = x^2 - 16x$?
3. How is the graph of $y = 2x^2 + 5$ different from the basic parabola? What are the coordinates of its vertex?
4. Compare the graphs of $y = x^2 - 25$ and $y = 25 - x^2$.

Lesson 13 Completing the Square

Activity 1 Squares of Binomials

1. a. Write a formula for the square of a binomial:

$$\boxed{(x+p)^2 = \qquad\qquad\qquad\qquad\qquad\qquad}$$

Notice that the constant term of the trinomial is ____________ , and the coefficient of the linear term (or x-term) is ____________ .

b. Now we'll reverse the process. Add a constant to the expression that will turn it into a perfect square:

$x^2 - 16x +$ __________ $= (x$_____________$)^2$

(Do you see how to find the constant? Here's what you do:)

First find p : $2p = -16$, so $p = \frac{1}{2}(-16) =$

Then find p^2 : $p^2 =$

We can turn any quadratic expression into a perfect square by adding the correct constant. This process is called **completing the square**.

2. Complete the square and write the result as the square of a binomial.

a. $x^2 + 8x +$ ________ $=$

b. $x^2 - 14x +$ ________ $=$

For parts (**c**) and (**d**), do not use a calculator! Work with fractions.

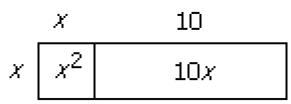
c. $x^2 + 3x +$ __________ $=$

d. $x^2 - \dfrac{5}{2}x +$ __________ $=$

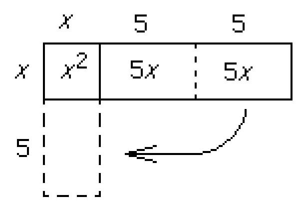
Activity 2 We can visualize completing the square using rectangles.

1. Study the diagram that illustrates completing the square for $x^2 + 10x$:

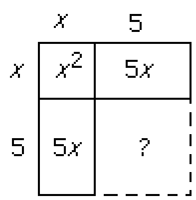
Step 1 Represent $x^2 + 10x$ as a rectangle



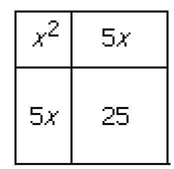
Step 2 Take $\frac{1}{2}$ of the linear term, and rearrange the pieces



Step 3 What is the missing piece?



Step 4 $x^2 + 10x + 25 = (x+5)^2$



2. In the space below, draw a similar set of diagrams to illustrate completing the square for $x^2 + 16x$.

Activity 3 Solving Quadratic Equations by Completing the Square

We use the technique of completing the square to solve quadratic equations. There are two parts to this method:

> **1.** Create a perfect square
> **2.** Use extraction of roots

1. a. Follow the steps to solve $x^2 - 6x - 16 = 0$

Move the constant term to the other side of the equation:

$$x^2 - 6x \qquad = 16$$

Complete the square on the left side:

$$p = \frac{1}{2}(-6) = \underline{\qquad} \quad \text{and} \quad p^2 = (-3)^2 = \underline{\qquad}$$

Add 9 to *both* sides of the equation:

$$x^2 - 6x + \mathbf{9} = 16 + \mathbf{9}$$

Write the left side as the square of a binomial:

$$(\underline{\qquad\qquad})^2 = 25$$

Use extraction of roots to find the solutions;
take the square root of both sides:

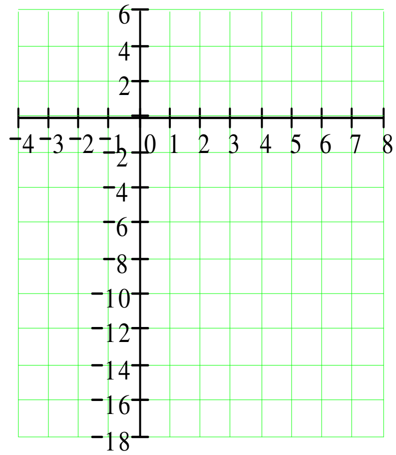
$$x - 3 = \underline{\qquad} \quad \text{or} \quad x - 3 = \underline{\qquad}$$
$$x = \underline{\qquad} \quad \text{or} \quad x = \underline{\qquad}$$

The solutions are _____ and _____.

b. Graph the parabola

$$y = x^2 - 6x - 16$$

on your calculator, and copy the graph onto the grid. What are the x-intercepts of the graph?



2. Solve $x^2 - 8x - 4 = 0$ by completing the square. (This equation *cannot* be solved by factoring!)

Move the constant term to the right side:

Complete the square: $p = \frac{1}{2}(-8) = \underline{\qquad\qquad}$ $p^2 = \underline{\qquad\qquad}$

Add p^2 to *both* sides:

Write the left side as the square of a binomial, and simplify the right side:

Extract roots to obtain *two* solutions:

Use a calculator to find decimal approximations for each solution. Round to thousandths:

3. Our method for completing the square only works if the coefficient of x^2 is 1.

> **If the lead coefficient is *not* 1, we must divide each term of the equation by the lead coefficient.**

a. Solve $4x^2 - 12x - 1 = 0$ by completing the square.

Divide each term by 4:

Move the constant to the right:

Complete the square on the left : $p = \frac{1}{2}(-3) =$ ______ , $p^2 =$ _______

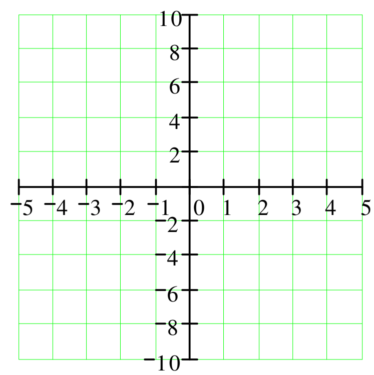
Add p^2 to *both* sides:

Write the left side as a perfect square;
simplify the right side.

Extract roots to obtain *two* solutions:

Use a calculator to find decimal approximations
for each solution. Round to thousandths:

b. Graph $y = 4x^2 - 12x - 1$ in the standard window.
Sketch the graph on the grid.
What are the x-intercepts of the graph?



Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Add a constant to $x^2 + bx$ to create a perfect square
- Solve quadratic equations by completing the square

Check Your Understanding

1. Name three algebraic methods for solving a quadratic equation.
2. Give an example of a quadratic trinomial that is the square of a binomial.
3. In Activity 3, what were the two parts named for solving an equation by completing the square?
4. What is wrong with the following solution?

$$\begin{aligned} 2x^2 - 8x + 3 &= 0 \\ 2x^2 - 8x + 16 &= -3 + 16 \\ (2x - 4)^2 &= 13 \\ x &= \frac{4 \pm \sqrt{13}}{2} \end{aligned}$$

Lesson 14 Quadratic Formula

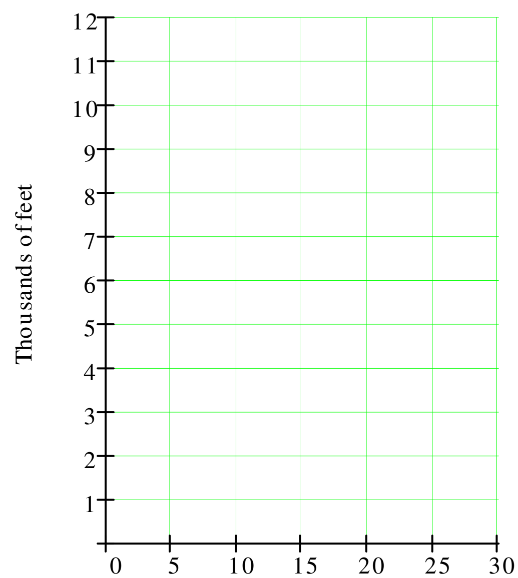
Activity 1
A skydiver jumps out of an airplane at 11,000 feet. While she is in free-fall, her altitude in feet t seconds after jumping is given by

$$h = -16t^2 - 16t + 11{,}000$$

a. Complete the table of values. (Use the **Table** feature of your calculator.)

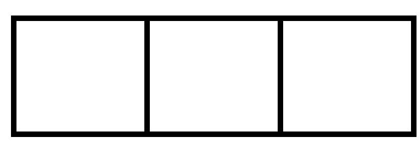
t	0	5	10	15	20	25	30
h							

b. Graph the equation.
c. Write and solve an equation to answer the question: If the skydiver must open her parachute at an altitude of 1000 feet, how long can she free-fall?



d. Locate the corresponding point on your graph.

Activity 2
A dog trainer has 100 meters of chain link fence. She wants to enclose a total area of 250 square meters in three pens of equal size, as shown.



We will find the length and width of each pen.
a. Let l and w represent the length and width of each pen. Label the figure. (How many l's and w's do you have?)
b. Write an equation in l and w about the amount of chain link fence. (**Hint:** Is this an area or a perimeter?)

c. Solve your equation for l in terms w.

d. Write an equation in terms of l and w for the total area enclosed.

e. Substitute your expression for l from part (c) into your equation from part (d).

f. Solve your equation from part (d). (There are two solutions!)

g. Find the dimensions of each pen. Round your answers to hundredths. ("Dimensions" means length and width. There are two different solutions to the problem.)

Activity 3 Summary: Solving Quadratic Equations

1. a. List four algebraic methods for solving quadratic equations:

(1)
(2)
(3)
(4)

b. Which method(s) work on *any* quadratic equation?

Which method is fastest if one side of the equation is a perfect square?

2. For each equation, name the easiest method for solving, then solve.

a. $x^2 - 7 = 0$

b. $x^2 = 7x$

c. $x^2 - 7x = 8$

d. $(x - 7)^2 = 8$

e. $x^2 = 7x - 8$

f. $x^2 - 8x = 4$

Activity 4 Deriving the Quadratic Formula

1. Use completing the square to solve the equation $x^2 + bx + c = 0$ for x in terms of b and c.

Move the constant term to the right side.

Complete the square on the left side.

$p =$ $p^2 =$

Write the left side as a perfect square;
simplify the right side.

Extract roots.

2. Use completing the square to solve $ax^2 + bx + c = 0$ for x in terms of a, b, and c.

Move the constant term to the right side;
divide both sides by the lead coefficient.

Complete the square on the left side.

$p =$ $p^2 =$

Write the left side as a perfect square;
simplify the right side.

Extract roots.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Use the quadratic formula to solve quadratic equations
- Use the discriminant to determine the number of x-intercepts
- Solve quadratic equations for one variable in terms of the others
- Solve problems arising from quadratic models

Check Your Understanding

1. In Activity 1c, what value did you substitute for h? Why? When is $h = 11,000$?

2. In Activity 2, did you solve an equation about area or about perimeter to find the values of w? How did you use the other quantity in your solution?
3. In Activity 2f, you found two solutions to the equation. Are these values the dimensions of each pen?
4. In Activity 4, what was different about the procedure in part 2 from the procedure in part 1?

Lesson 15 The Vertex

Activity 1 The Vertex

1. a. How do you know that the graph of $y = (x-2)^2 - 6$ is a parabola?

b. Does the parabola open up or down? Why?

c. What is the smallest y-value on the graph of $y = (x-2)^2 - 6$?
(Hint: What is the smallest value that $(x-2)^2$ can have?)

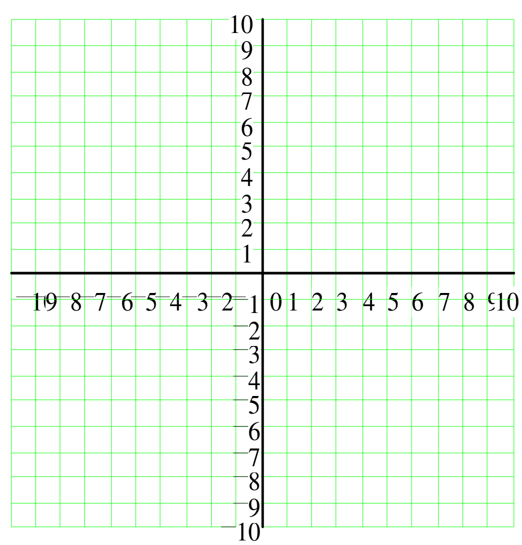
d. Which x-value gives us the smallest y-value? Why?

e. Graph the equation in the "friendly" window and verify your answers.

2. a. Find the vertex of the graph of $y = 3(x-2)^2 - 6$.

b. Write the equation in part (a) in standard form.

3. a. Find an equation for a parabola whose vertex is $(4, -7)$.



b. Sketch your parabola at right.

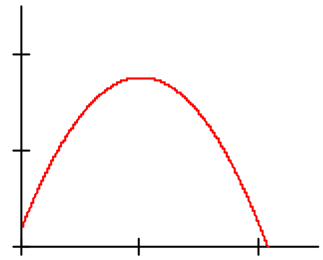
c. Find an equation for the parabola in part (a) if one point on the graph is $(2, 9)$.

d. Sketch the parabola from part (c) on the same grid.

Activity 2 Vertex Form

The batter in a softball game hits the ball when it is 4 feet above the ground. The ball reaches the greatest height, 35 feet, directly above the head of the left-fielder, who is 200 feet from home plate.

a. The path of the ball is shown at right. Label the y-intercept and the vertex with their coordinates.



b. Write an equation for the height of the ball in terms of the horizontal distance it has traveled.

c. Find the height of the ball when it reaches the left field wall, which is 375 feet from home plate. If the wall is 10 feet tall, did the batter hit a home run (did the ball go over the wall)?

Activity 3 Maximum or Minimum Value

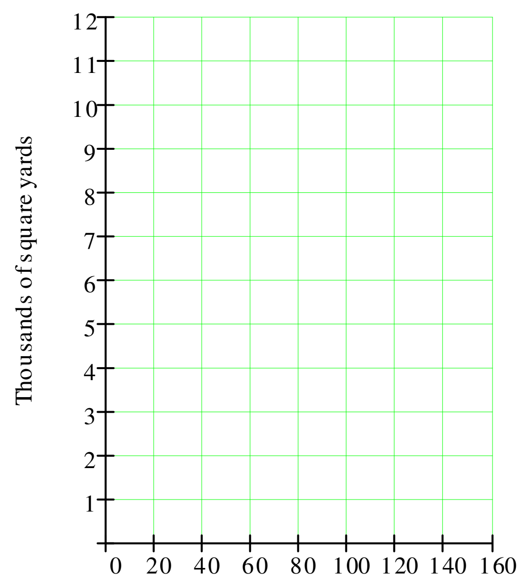
A farmer plans to fence a rectangular grazing area along a river using 300 yards of fence.

a. Draw a sketch illustrating the grazing area, and label its length and width.

b. Let w stand for the width of the rectangle, and write an expression for the length l in terms of the width.

c. Write an equation for the area A of the grazing land in terms of w.

d. Graph the equation.



e. Use algebra to answer the questions:
What is the largest area the farmer can enclose?

What are the dimensions of the largest area?

Activity 4 Revenue

The local theater group sold tickets to its opening night performance for \$5 and drew an audience of 100 people. The next night they reduced the ticket price by \$0.25 and 10 more people attended; that is, 110 people bought tickets at \$4.75 apiece. In fact, for each \$0.25 reduction in ticket price, 10 additional tickets can be sold.

a. Complete the table.

Number of Price Reductions	Price of Ticket	Number of Tickets Sold	Total Revenue
0	5.00	100	500
1	4.75	110	522.50
2			
3			
5			
8			
10			
11			

b. Let x represent the Number of price reductions, as in the first column of the table. Write algebraic expressions in terms of x for:
The Price of a Ticket after x price reductions:

Price $=$

The Number of Tickets Sold at that price:

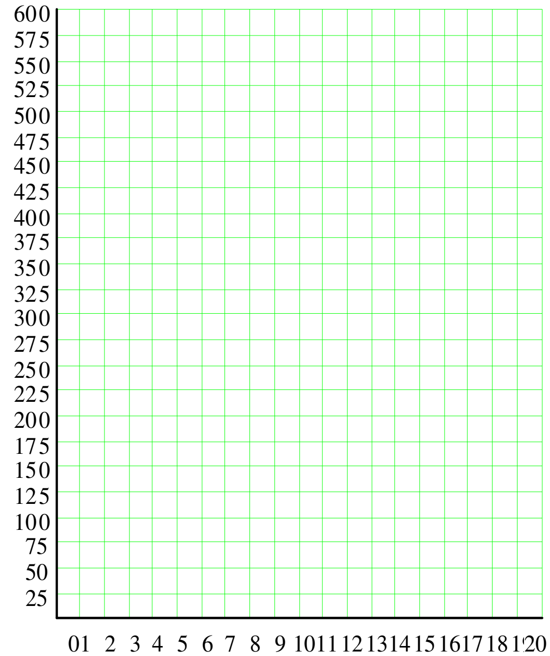
Number $=$

The total revenue from ticket sales:

Revenue $=$

c. Enter your expressions into the calculator:
$\mathbf{Y_1}$ = the price of a ticket,
$\mathbf{Y_2}$ = the number of tickets sold,
$\mathbf{Y_3}$ = the total revenue.
Use the **Table** feature to verify that your algebraic expressions agree with your table from part (a).

d. Use your calculator to graph your equation for total revenue in terms of x. Sketch the graph on the grid.



e. What is the maximum revenue possible from ticket sales?

What is the value of x at that revenue?

f. Use your formulas for $\mathbf{Y_1}$ and $\mathbf{Y_2}$ to answer the questions:
What price should the theater group charge for a ticket to generate the maximum revenue?

How many tickets will they sell at that price?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Find the vertex of a parabola
- Find the maximum or minimum value of a quadratic model
- Write a quadratic equation in vertex form by completing the square
- Find an equation for a parabola given the vertex and one other point
- Write a quadratic model for revenue

Check Your Understanding

1. The height of a golf ball in meters is given by $h = -4.9t^2 + 20t$.
  a. Explain how to find out when the ball hits the ground.
  b. Explain how to find the greatest height the ball reaches.

2. Explain why you need another point besides the vertex to find the equation of a parabola.

3. Explain why revenue does not increase indefinitely as you increase price.

4. What is the smallest value that y can have if $y = 2(x-3)^2 + 6$? What value of x produces that smallest y-value?

Lesson 16 Curve Fitting

Activity 1 Curve Fitting

You are driving at 60 miles per hour when you step on the brakes. Find a quadratic formula, $d = at^2 + bt + c$, for the distance in feet that your car has traveled t seconds after braking.

t (sec)	1	2	3	4
d (feet)	81	148	210	240

Follow the steps to solve the problem:

Step 1 Use the first three data points from the table to write three equations

Step 2 Simplify the equations, and write a 3×3 system

Step 3 Solve the system

Step 4 State the solution to the problem

Activity 2 Finding the Equation of a Parabola

1. We now have three forms for the equation of a parabola:

standard form	$y = ax^2 + bx + c$
factored form	$y = a(x - r_1)(x - r_2)$
vertex form	$y = a(x - x_v)^2 + y_v$

Each of these forms is useful for highlighting different properties of the parabola.

a. What is the y-intercept of the graph of $y = 3x^2 - 5x + 8$?

b. What are the x-intercepts of the graph of $y = -2(x-3)(x+5)$?

c. What is the vertex of the graph of $y = 4(x+2)^2 - 5$?

How can we find an equation for a parabola?

1. If we know the x-intercepts of the parabola, then we use the factored form.
2. If we know the vertex, we use the vertex form.
3. In each case, we must know *one other point* on the parabola in order to find the value of a.

2. Choose one of the three forms to solve each problem about parabolas:

Problem 1

a. Write an equation for a parabola that has x-intercepts at $(2,0)$ and $(-3,0)$. (There are many possible answers.)

b. Write an equation for another parabola that has the same x-intercepts.

c. Write an equation for the parabola that has x-intercepts at $(2,0)$ and $(-3,0)$ and y-intercept at $(0,3)$.

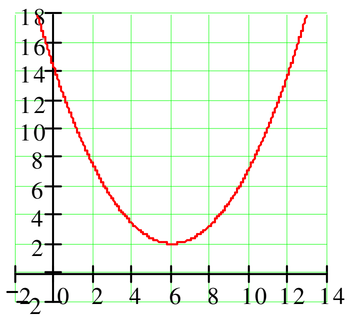
Problem 2

a. Write an equation for a parabola whose vertex is the point $(-2, 6)$. (Many answers are possible.)

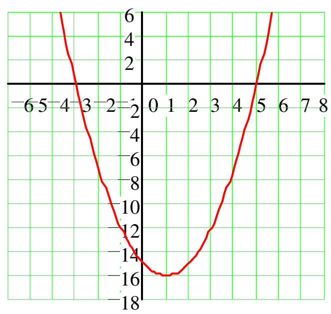
b. Find the value of a if the y-intercept of the parabola in part (a) is 18.

Problem 3
Find an equation for each parabola. Use the vertex form or the factored form of the equation, whichever is more appropriate.

a.

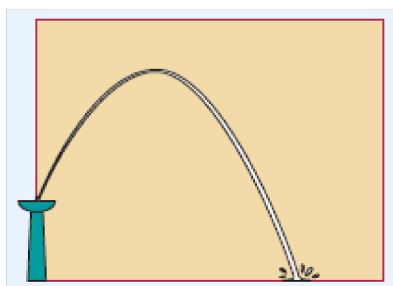


b.



Activity 3 Curve-Fitting with the Vertex Form

Francine is designing a synchronized fountain display for a hotel in Las Vegas. For each fountain, water emerges in a parabolic arc from a nozzle three feet above the ground. Francine would like the vertex of the arc to be eight feet high, and two feet horizontally from the nozzle.



a. Choose a coordinate system with its origin at the base of the fountain. Label the coordinates of two points on the path of the water.

b. Write an equation for the path of the water.

c. How far from the base of the nozzle will the stream of water hit the ground?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Use Gaussian elimination to fit a parabola to three points
- Use a calculator for quadratic regression
- Choose the best method to find the equation for a parabola

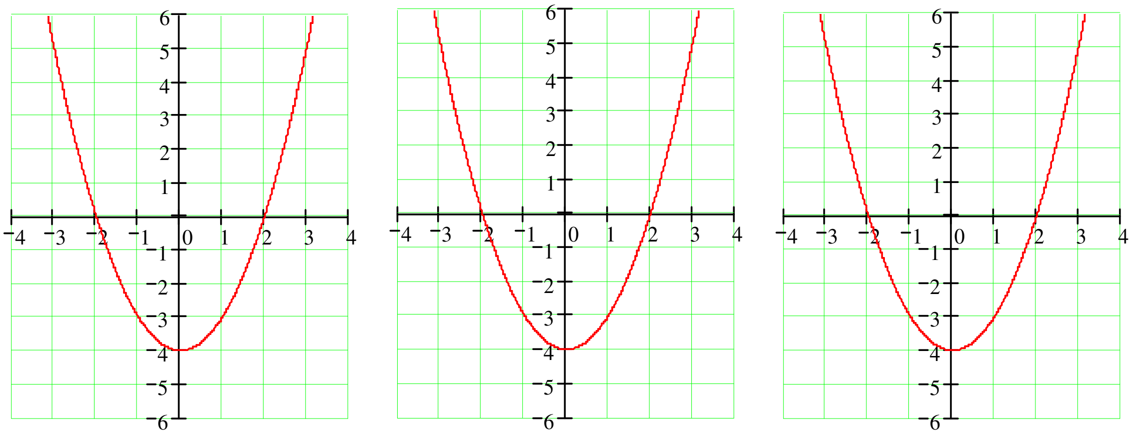
Check Your Understanding

1. What were the variables in your 3×3 system in Activity 1?
2. If you use the last three points in the table instead of the first three points, will you get the same regression equation? Why or why not?
3. Name three forms for quadratic equations.
4. Why do we need to know a second point besides the vertex when using the vertex form to find the equation of a parabola?

Lesson 17 Quadratic Inequalities

Activity 1 Solving Quadratic Inequalities Graphically

Here are three copies of the graph of $y = x^2 - 4$



a. On the first graph, we'll solve the equation $x^2 - 4 = 0$ (Remember that $y = x^2 - 4$.)

Step 1 Find two points on the graph with $y = 0$. Put dots there.
Step 2 What are the x-coordinates of those points?
Step 3 The solutions are:

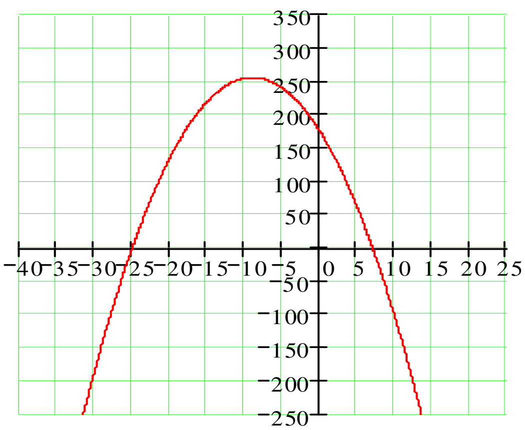
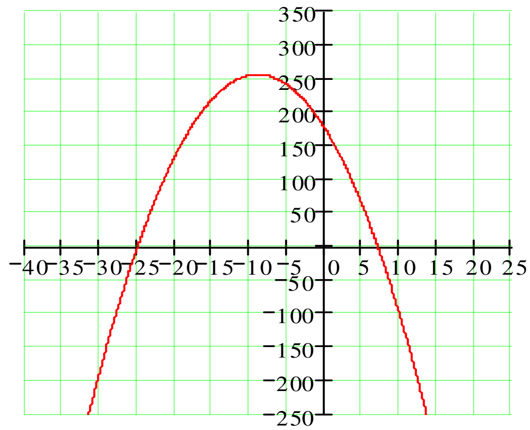
b. On the second graph, we'll solve the inequality $x^2 - 4 < 0$

Step 1 Mark all points on the graph that have $y < 0$.
Step 2 On the x-axis, mark the x-coordinates of all those points.
Step 3 Write an inequality to describe the portion of the x-axis marked.

c. On the third graph, we'll solve the inequality $x^2 - 4 > 0$

Step 1 Mark all points on the graph that have $y > 0$.
Step 2 On the x-axis, mark the x-coordinates of all those points.
Step 3 Write two inequalities to describe the portion of the x-axis marked.

2. Here are two copies of the graph of $y = 175 - 18x - x^2$

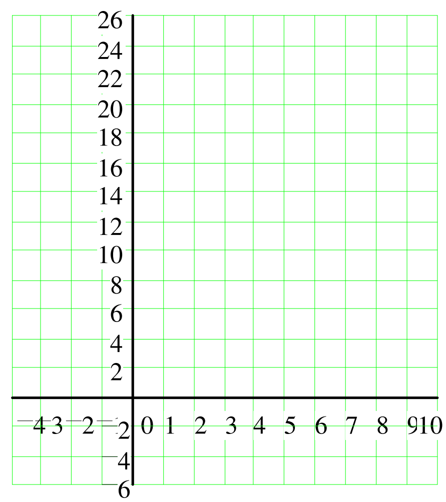
a. Use the first graph to solve $175 - 18x - x^2 = 0$
(Hint: Notice that one of the x-intercepts is $x = -25$. Think of the factored form of the equation. What is the other factor?)

$$-(x+25)(x - \underline{\hspace{2cm}}) = 0$$

b. Use the second graph to solve $175 - 18x - x^2 < 0$
Step 1 Mark all points on the graph that have $y < 0$.
Step 2 On the x-axis, mark the x-coordinates of all those points.
Step 3 Write two inequalities to describe the portion of the x-axis marked.

3. Solve the inequality $20 + 4x - x^2 \leq 8$, and write your answer with interval notation.

a. Rewrite the inequality so that the right side is zero.



b. Graph the equation $y = 12 + 4x - x^2$

y-intercept:

x-intercepts: Solve $12 + 4x - x^2 = 0$

Vertex: $x_v = \dfrac{-b}{2a} =$

$y_v =$

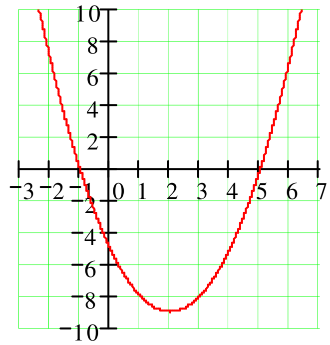
c. Use the graph to solve the inequality. Write the solution in interval notation:

Activity 2   Solving Inequalities Algebraically

1. a. Use the graph to solve

$$x^2 - 4x - 5 \geq 0$$

Show the solution on the graph.
Write your answer in interval notation:



$y = x^2 - 4x - 5$

b. Notice that, in order to solve the inequality, we don't need a totally accurate graph; we really only need to know two things.

To solve a quadratic inequality,
$ax^2 + bx + c < 0$ or $ax^2 + bx + c > 0$
answer two questions:
(1) What are the x-intercepts of the related graph?
(2) Does the parabola open up or down?

2. Follow the steps to solve $4x^2 + 8x - 5 \leq 0$

Step 1 Find the x-intercepts of $y = 4x^2 + 8x - 5$

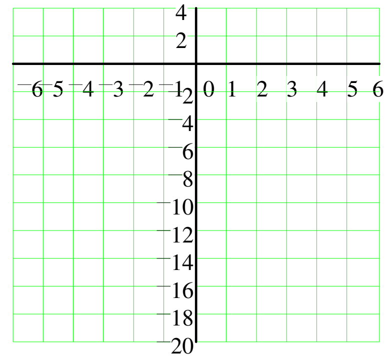
Step 2 Does the graph of $y = 4x^2 + 8x - 5$ open up or down? Make a rough sketch of the graph, and label the x-intercepts.

Step 3 Use the graph to solve the inequality. Write your answer in interval notation.

Remember that you may need to use extraction of roots or the quadratic formula to find the x-intercepts of a parabola.

3. Solve $x^2 > 18$

Step 1 Write the inequality in standard form.

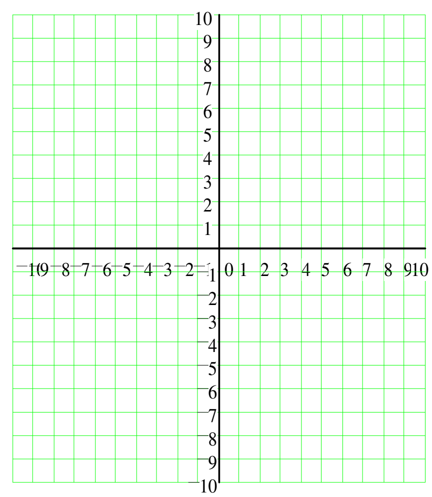


Step 2 Find the x-intercepts of the corresponding graph.

Step 3 Make a rough sketch of the graph. Use the graph to solve the inequality. Write your answer with interval notation

4. Solve the inequality $10 - 8x + x^2 > 4$

Step 1 Write the inequality in standard form.



Step 2 Find the x-intercepts of the corresponding graph. (Use the quadratic formula.)

Step 3 Make a rough sketch of the graph. Use the graph to solve the inequality. Write your answer with interval notation

Wrap-Up

In this Lesson, we worked on the following skills and goals related to quadratic models:

- Write compound inequalities
- Use interval notation
- Solve quadratic inequalities graphically
- Solve quadratic inequalities algebraically
- Write a quadratic model for revenue

Check Your Understanding

1. In Activity 1, we saw that for any value of x, the value of $x^2 - 4$ is either _______ , ______, or ______ .

2. Suppose you have a point P with y-coordinate 8 on the graph of an equation. How do you find the x-value that produces $y = 8$ in the equation?

3. When you solve a quadratic inequality with a graph, how do you find the boundary points of the solution interval(s)?

4. Use a graph to explain why the inequality $x^2 + 1 < 0$ has no solution.

Lesson 18 Functions

Activity 1 Epidemics

The spread of a contagious disease can devastate a confined population. For example, in the early sixteenth century Spanish troops introduced smallpox into the Aztec population in Central America, and the resulting epidemic contributed significantly to the fall of Montezuma's empire.

Suppose that an outbreak of cholera follows severe flooding in an isolated town of 5000 people. Initially (on Day 1), 40 people are infected. Every day after that, 25% of those still healthy fall ill.

Day	New Patients	Total Infected	Number Healthy
1	40	40	4960
2			
3			
4			
5			
6			
7			
8			
9			
10			

1. At the start of the second day (Day 2), how many people are still healthy?

How many fall ill during Day 2?

What is the total number of people infected at the end of Day 2?

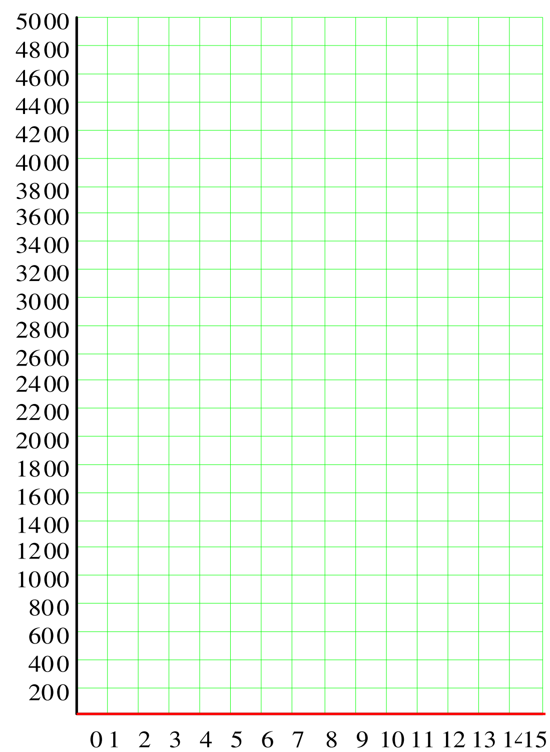
How many people are still healthy at the end of Day 2?

2. Fill in the rest of the table for ten days. (Round off decimal results to the nearest whole number.)

3. Use the third column of the table to plot the total number of infected residents, I, against time, t. Connect your data points with a smooth curve.

4. Do the values of I approach some largest value? Draw a dotted horizontal line at that value of I. Will the values of I ever exceed that value? Why or why not?

5. What is the first day on which at least 50% of the population is infected?



6. Look back at the table. What is happening to the number of new infected residents each day as time goes on? How is this phenomenon reflected in the graph? How would your graph look if the number of new patients every day were a constant?

7. Summarize your work: In your own words, describe how the number of residents infected with cholera changes with time. Include a description of your graph. Is I a function of t? Why or why not?

Activity 2 Function Notation

Data indicate that U.S. women are delaying having children longer than their counterparts 50 years ago. The table shows $P = f(t)$, the percent of 20-24 year old women in year t who had not yet had children. (Source: U.S. Dept of Health and Human Services)

Year, t	1960	1965	1970	1975	1980	1985	1990	1995	2000
% of Women, P	47.5	51.4	47.0	62.5	66.2	67.7	68.3	65.5	66.0

a. How does the table support the assumption that P is a function of t? Does this assumption make sense for the context of the problem?

b. Evaluate $f(1985)$ and explain what it means.

c. Estimate a solution to the equation $f(t) = 68$ and explain what it means.

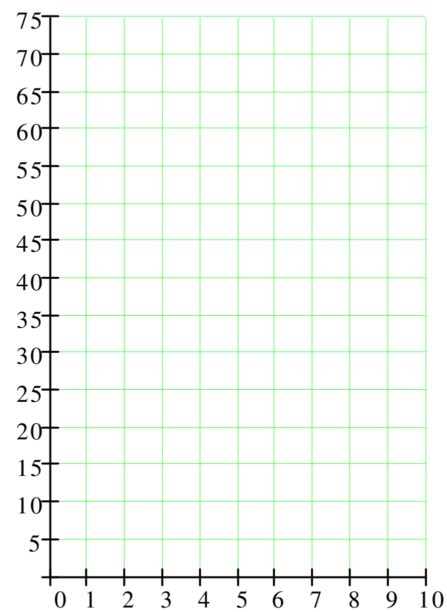
d. In 1997, 64.9% of 20-24 year old women had not yet had children. Write an equation with function notation that states this fact.

Activity 3 Functions and Graphs

The table shows data obtained while heating a solid sample of stearic acid, a waxy solid used in making candles, soap, and some plastics. Heat was applied at a constant rate throughout the experiment.

Time (min)	0	0.5	1.5	2	2.5	3	4	5	6	7	8	8.5	9	9.5	10
Temp (°C)	19	29	40	48	53	55	55	55	55	55	55	64	70	73	74

a. Plot the data with time on the horizontal axis and temperature on the vertical axis. Is temperature a function of time? Why or why not?



b. Choose variables and use function notation to state that temperature is a function of time. Label the axes on your graph.

c. Is temperature a **linear** function of time? Why or why not? Describe the temperature as a function of time.

d. Energy (in the form of heat) is required to raise the temperature of a substance, and it is also needed to melt a solid substance to a liquid. By analyzing the graph, what do you think is the melting point of stearic acid? How long did it take the sample to melt?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Recognizing functions
- Evaluating functions defined by tables, graphs, or equations
- Using function notation

Check Your Understanding

1. In Activity 1, what is the input variable, and what is the output variable?

2. In Activity 2, can you find a formula for P in terms of t? Does this mean that P is not a function of t? Why or why not?

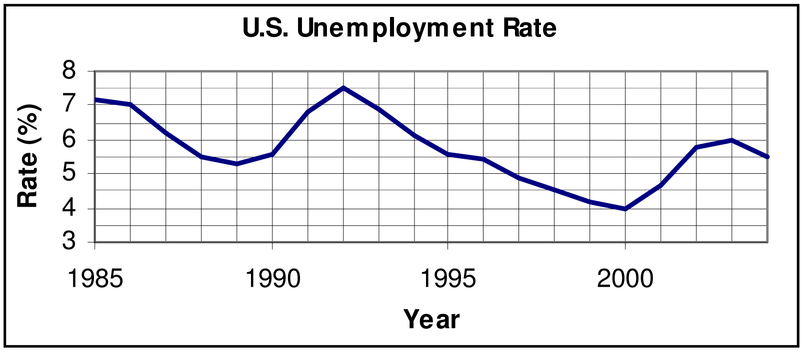
3. In Activity 3, is time a function of temperature? Why or why not?

4. In Activity 3, how can you estimate the temperature of the sample at times not listed in the table?

Lesson 19 Graphs of Functions

Activity 1 The Graph of a Function

The graph shows the U.S. unemployment rate as a function of time, $U = f(t)$, for the years 1985-2004. (Source: U.S. Bureau of Labor Statistics)



a. Evaluate $f(1993)$, and explain its meaning for this problem.

b. When did the unemployment rate reach its highest value, and what was its highest value? Write your answer with function notation.

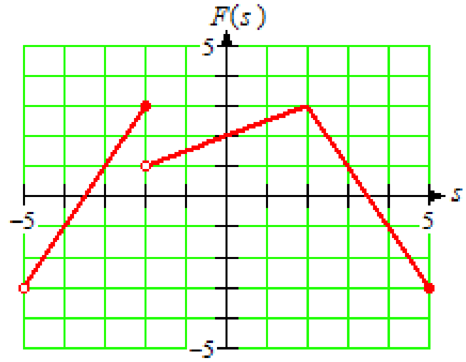
c. When did the unemployment rate fall to its lowest value, and what was its lowest value? Write your answer with function notation.

d. Solve the equation $f(t) = 4.5$, and explain its meaning for this problem.

e. Over what intervals was the unemployment rate increasing?

Activity 2 Functions Defined by Graphs

The figure shows the graph of the function $y = F(s)$. A solid dot means that the point is part of the graph, and an open circle means that the point is not part of the graph.



a. Find $F(-3)$, $F(-2)$, and $F(2)$.

b. For what value(s) of s is $F(s) = -1$?

c. Find the maximum value of $F(s)$. For what value(s) of s does F take on its maximum value?

d. Find the minimum value of $F(s)$. For what value(s) of s does F take on its minimum value?

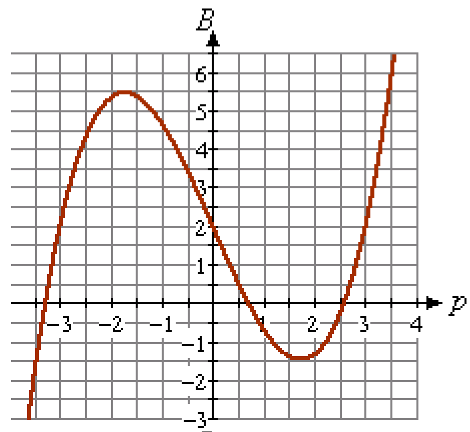
Activity 3   Solving Equations and Inequalities

Each figure shows a graph of

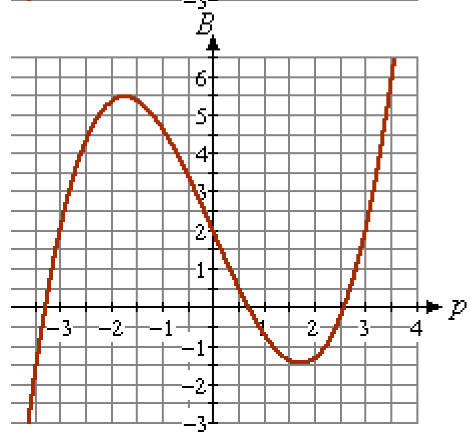
$$B = f(p) = \frac{1}{3}p^3 - 3p + 2$$

Use the graph to solve the following equations and inequalities. Show your work on the graph. Write your answers in interval notation.

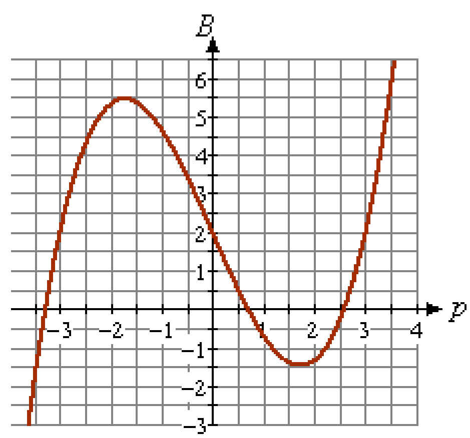
a. $\frac{1}{3}p^3 - 3p + 2 = 6$



b. $\frac{1}{3}p^3 - 3p + 2 = 2$



c. $\frac{1}{3}p^3 - 3p + 2 < 2$



Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Use function notation to label points on a graph
- Interpret the coordinates of a point on the graph of a function
- Construct the graph of a function
- Use the vertical line test
- Solve an equation or inequality graphically

Check Your Understanding

1. In Activity 1, what is the input variable, and what is the output variable?
2. In Activity 1d, how did you solve the equation?
3. Is it possible for a function to have more than one maximum value? Is it possible for a function to take on its maximum value at more than one point?
4. In Activity 3, how do you know that the graph represents a function?

Lesson 20 Some Basic Graphs

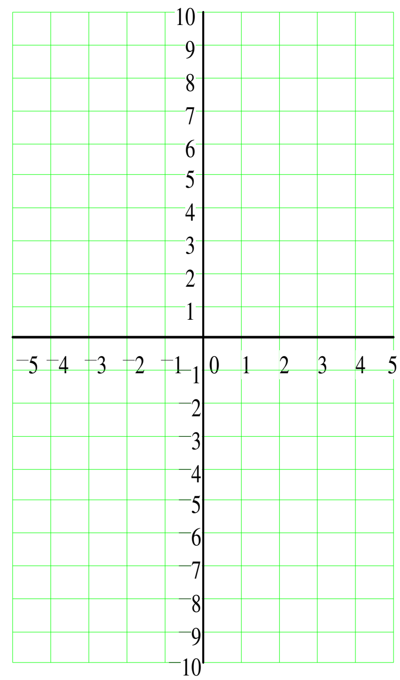
Activity 1 Some Powers and Roots

1. a. Complete the table of values for the two functions

$$f(x) = x^2 \text{ and } g(x) = x^3$$

b. Graph both functions on the grid. Pay particular attention to the curvature of the graphs near the origin.

x	$f(x) = x^2$	$g(x) = x^3$
-3		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
3		



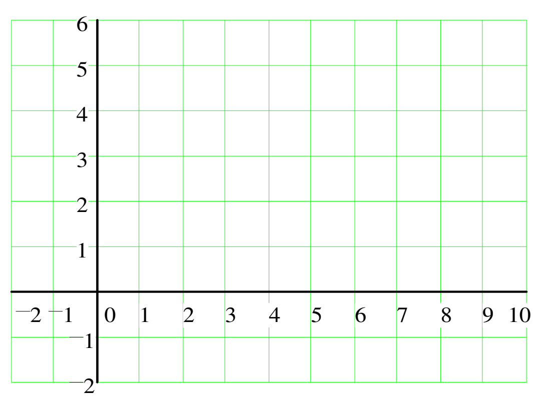
2.a. Complete the tables for the functions

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = \sqrt[3]{x}$$

(Round your answers to two decimal places.)

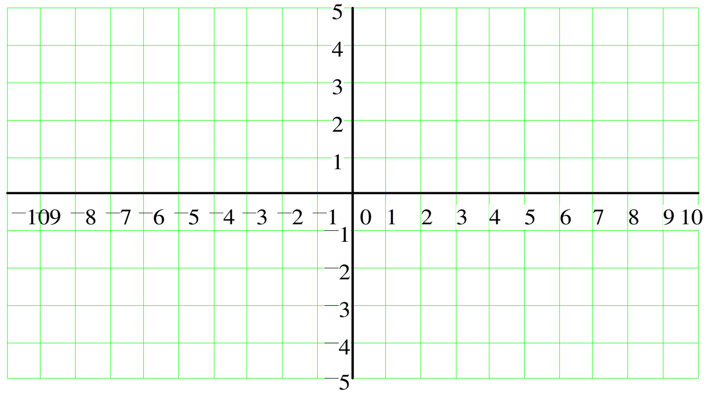
x	$f(x) = \sqrt{x}$
0	
$\frac{1}{2}$	
1	
2	
3	
4	
5	
7	
9	

b. Graph each function on the grid.



$f(x) = \sqrt{x}$

x	$g(x) = \sqrt[3]{x}$
-8	
-4	
-1	
$-\frac{1}{2}$	
0	
$\frac{1}{2}$	
1	
4	
8	



$g(x) = \sqrt[3]{x}$

Activity 2 Asymptotes

1. a. Complete the table for the functions

$$f(x) = \frac{1}{x} \text{ and } g(x) = \frac{1}{x^2}$$

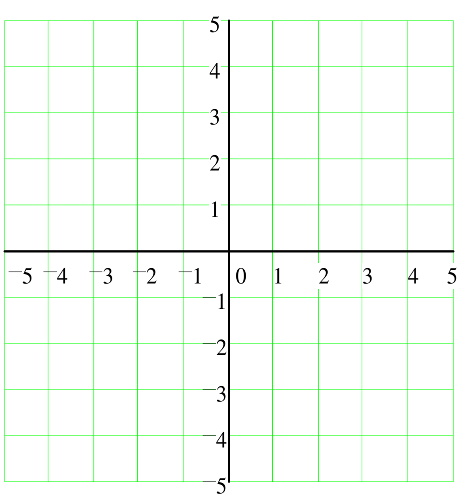
What is true about the y-values at $x = 0$?

x	$f(x) = \frac{1}{x}$	$g(x) = \frac{1}{x^2}$
-4		
-3		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
3		
4		

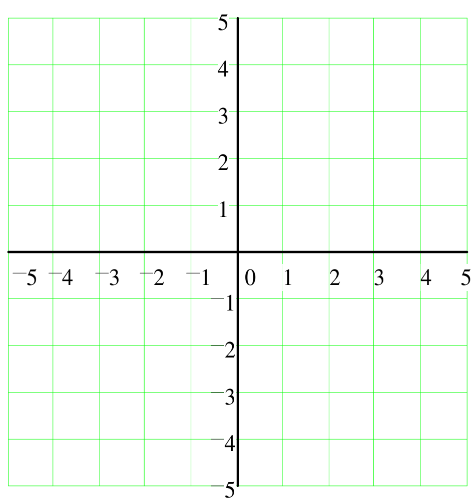
b. Plot the points from the table and connect them with smooth curves.

2. a. As x increases through larger and larger values, what happens to the values of $y = \frac{1}{x}$?

Extend your graph to reflect your answer.



$f(x) = \frac{1}{x}$



$g(x) = \frac{1}{x^2}$

b. What happens to y as x decreases through larger and larger negative values (that is, for $x = -5, -6, -7, \dots$)?
Extend your graph for these x-values.

As the values of x get larger in absolute value, the graph approaches the x-axis. However, because $\frac{1}{x}$ never *equals* zero for any x-value, the graph never actually touches the x-axis. We say that the x-axis is a **horizontal asymptote** for the graph.

3. Repeat part **2** for the graph of $g(x) = \frac{1}{x^2}$.

4. a. Next we'll examine the graphs near $x = 0$. Evaluate $f(x) = \frac{1}{x}$ for several x-values close to zero and complete the table below. What happens to the values of $y = \frac{1}{x}$ as x approaches zero?

Extend your graph to reflect your answer.

x	$f(x) = \frac{1}{x}$
-2	
-1	
-0.1	
-0.01	
-0.001	

x	$f(x) = \frac{1}{x}$
2	
1	
0.1	
0.01	
0.001	

As x approaches zero from the left (through negative values), the y-values decrease towards $-\infty$. As x approaches zero from the right (through positive values), the y-values increase towards ∞. The graph approaches but never touches the vertical line $x = 0$ (the y-axis.) We say that the graph has a **vertical asymptote** at $x = 0$.

b. Repeat part (a) for the graph of $g(x) = \frac{1}{x^2}$.

x	$g(x) = \frac{1}{x^2}$
-2	
-1	
-0.1	
-0.01	
-0.001	

x	$g(x) = \frac{1}{x^2}$
2	
1	
0.1	
0.01	
0.001	

5. The functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ are examples of **rational functions**. Verify both graphs with your graphing calculator. Use the window

$$\text{Xmin} = -4 \quad \text{Xmax} = 4$$
$$\text{Ymin} = -4 \quad \text{Ymax} = 4$$

6. State the intervals on which each graph is increasing.

7. Write a few sentences comparing the two graphs.

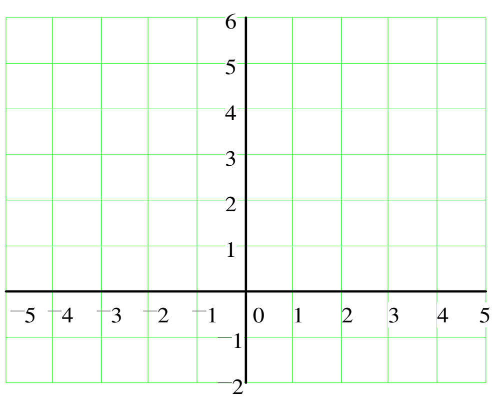
Activity 3 Absolute Value

1.a. Complete the table of values for the two functions

$$f(x) = x \text{ and } g(x) = |x|$$

b. Graph both functions on the grid.

x	$f(x) = x$	$g(x) = \lvert x \rvert$
-4		
-2		
-1		
$-\frac{1}{2}$		
0		
$\frac{1}{2}$		
1		
2		
4		



$f(x) = x, \quad g(x) = |x|$

2. Verify both graphs with your graphing calculator. Your calculator uses the notation **abs** (x) instead of $|x|$ for the absolute value of x. Access the absolute value function through the **Catalog**: after opening the **Y=** screen, press **2nd** **0** to see the **Catalog**, then press **ENTER** for abs (x).

3. State the intervals on which each graph is increasing.

4. Write a few sentences comparing the two graphs.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Evaluate cube roots and absolute values
- Graph eight basic functions
- Use horizontal and vertical asymptotes for graphing

Check Your Understanding

1. In Activity 1, describe the differences between the graphs of $y = x^2$ and $y = x^3$.

2. What connection do you see between the graphs of $y = x^2$ and $y = \sqrt{x}$, and between the graphs of $y = x^3$ and $y = \sqrt[3]{x}$?

3. Explain how we use asymptotes to help us sketch a graph.

4. Which of the functions in this Lesson are undefined at $x = 0$?

Lesson 21 Variation

Activity 1 Direct Variation

1. Complete the statements about direct variation.

a. Definition: y is **proportional** to x, or **varies directly** with x, if the ratio $\frac{y}{x}$ is ______________ .

b. From this fact we can write a formula for y in terms of x: ____________ .

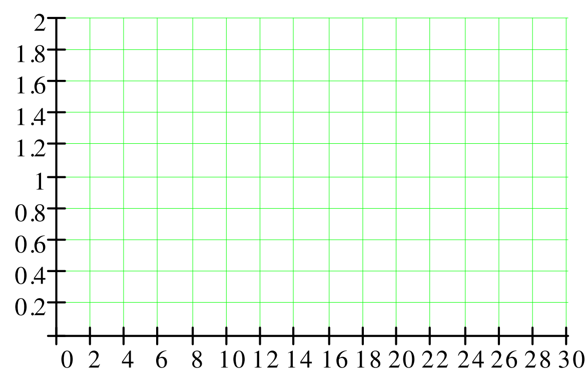
c. Use your answer to (**b**) to describe the graph of a direct variation:

d. The **constant of variation** is just the _____________ of the graph.

2. Delbert's credit card statement lists three purchases he made while on a business trip in the midwest. His company's accountant would like to know the sales tax rate on the purchases.

Price of item	18	28	12
Tax	1.17	1.82	0.78
Tax/Price			

a. Compute the ratio of the tax to the price of each item. Is the tax proportional to the price? What is the tax rate?



b. Express the tax, T, as a function of the price, p, of the item.

c. Sketch a graph of the function.

d. What is the slope of the graph?

3. Look back at the example above to answer the questions.

a. How can you recognize a direct variation from a table of values?

b. How can you find the constant of variation?

General strategy for variation problems:

Step 1 First, use the values given in the problem to find the constant of variation. Then you can write a formula for the function.

Step 2 Now use the formula to answer the questions.

4. The weight of an object on the moon varies directly with its weight on earth. A person who weighs 150 pounds on earth would weigh only 24.75 pounds on the moon.

a. Find a formula that gives the weight m of an object on the moon in terms of its weight w on earth.

b. Complete the table.

w	100	150	200	400
m				

c. Use the table to choose a suitable window and graph your function on your calculator. Sketch the graph in the space above, and label the window settings.

d. How much would a person weigh on the moon if she weighs 120 pounds on earth?

e. A piece of rock weighs 50 pounds on the moon. How much will it weigh back on earth?

f. Label the points on your graph that correspond to your answers for (**d**) and (**e**).

Activity 2 Direct Variation with a Power

1. Complete the statements about direct variation.

a. The formula for direct variation with a power is ____________________ .

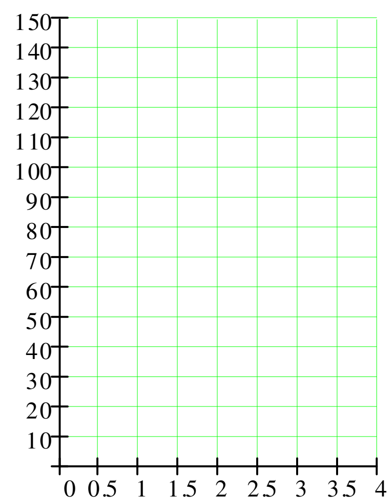
b. From that formula, we see that the ratio ________________ is constant.

c. The graph of a direct variation always passes through ______________ .

2. At constant acceleration from rest, the distance traveled by a racecar is proportional to the square of the time elapsed. The highest recorded road-tested acceleration is 0 to 60 miles per hour in 3.07 seconds, which produces the following data.

Time (seconds)	2	2.5	3
Distance (feet)	57.32	89.563	128.97
Distance/Time^2			

a. Compute the ratios of the distance traveled to the **square** of the time elapsed. What is the constant of proportionality?



b. Express the distance traveled, d, as a function of time in seconds, t.

c. Sketch a graph of the function.

3. Look back at the example above to answer the questions.

a. In part (**a**), note that you computed the ratio $\frac{d}{t^2}$ (**not** the ratio $\frac{d}{t}$). Why?

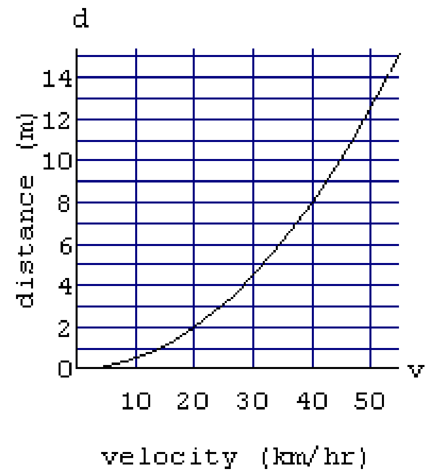
b. What sort of graph do you get in part (**c**)?

4. The faster a car moves, the more difficult it is to stop. The graph shows the distance, d, required to stop a car as a function of its velocity, v, before the brakes were applied.

a. Find a formula for d as a function of v.

Hints: 1) How do you know from the graph that $d = kv^2$ (instead of $d = kv$) ?

2) Use a point on the graph to find the value of k, and then write the formula.



b. How fast was the car moving if it took 50 meters to stop?

Activity 3 Inverse Variation

1. Complete the statements about inverse variation.

a. The formula for inverse variation is ________________ .

b. An inverse variation is an example of a __________________ function.

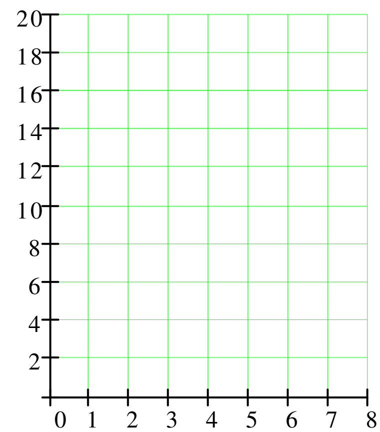
c. The graph of an inverse variation has a ____________________ at $x = 0$.

d. If y varies inversely with x, so that $y = \frac{k}{x}$, what is true about the product of the variables?

2. The marketing department for a paper company is testing wrapping paper rolls in various dimensions to see which shape consumers prefer. All the rolls contain the same amount of wrapping paper.

Width (feet)	2	2.5	3
Length (feet)	12	9.6	8
Length · Width			

a. Compute the product of the length and width for each roll of wrapping paper. What is the constant of inverse proportionality?



b. Express the length, L, of the paper as a function of the width, w, of the roll.

c. Sketch a graph of the function. Which basic graph does your answer resemble?

3. Ocean temperatures are generally colder at the greater depths. The table shows the temperature of the water as a function of depth.

a. How do you know from the table that this is an inverse variation?

Depth (m)	Temperature (° C)
100	30
200	15
300	10
400	7.5

b. Find a formula for T, temperature, in terms of D, depth.

c. What is the ocean temperature at a depth of 500 meters?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Recognize direct and inverse variation from a table of values
- Find the constant of variation and write an equation
- Sketch the graph of a direct or inverse variation
- Use scaling in inverse or direct variation

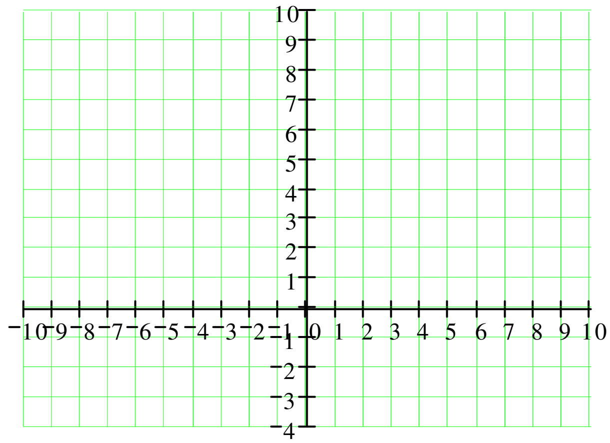
Check Your Understanding

1. How can you recognize direct variation from an equation, a graph, or a table of values?
2. What ratio should you compute to see if y varies directly with x^3?
3. Is every decreasing function an inverse variation? Explain.
4. How do you know that the table in Activity 3 represents inverse variation?

Lesson 22 Absolute Value Equations

Activity 1 Absolute Value Equations

1. Solving with a graph



a. Graph $y = |x - 3|$ on your calculator, and copy the graph onto the grid at right.

b. Sketch the horizontal line $y = 5$ on your graph.

c. We'll use the graph to solve the equation

$$|x - 3| = 5$$

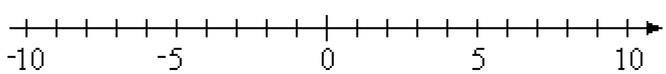
Find the points on the graph of $y = |x - 3|$ that have $y = 5$. (How many points are there?) What are the x-coordinates of those two points?

2. Solving with a number line

a. Translate the equation $|x - 3| = 5$ into a sentence about distance.

b. Locate 3 on the number line below, and find two points at a distance of 5 units from 3.

c. What are the x-coordinates of those two points?



3. Solving with algebra

a. Recall the definition of absolute value:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Now substitute $x - 3$ for x:

$$|x - 3| = \begin{cases} x - 3 & \text{if } x - 3 \geq 0 \\ -(x - 3) & \text{if } x - 3 < 0 \end{cases}$$

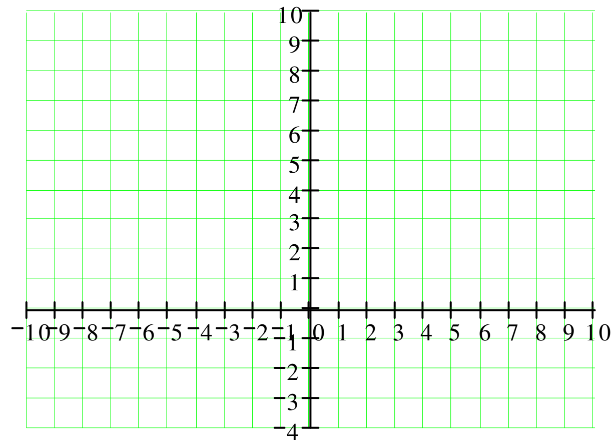
b. Thus, the equation $|x - 3| = 5$ has two parts:

$x - 3 = 5$ and $-(x - 3) = 5$

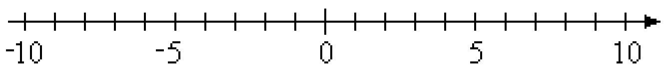
c. Solve each equation to get two solutions:

4. Solve the equation $|x+6|=2$ in three different ways:

a. By graphing $y=|x+6|$



b. By using a number line



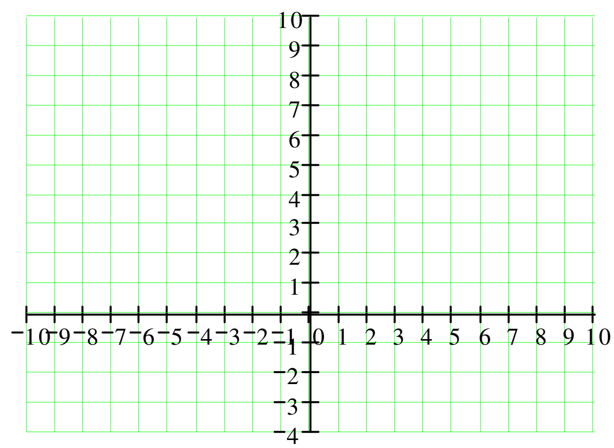
c. By using algebra

Activity 2 Absolute Value Inequalities

1. Solving with a graph

a. Graph $y=|x-3|$ on your calculator, and copy the graph onto the grid.
b. Sketch the horizontal line $y=5$ on your graph.
c. We'll use the graph to solve the inequality

$$|x-3|<5$$



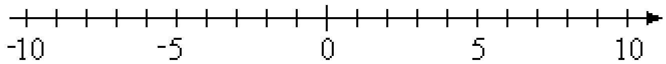
Shade the points on the graph of $y=|x-3|$ that have $y<5$.
Now shade the portion of the x-axis that corresponds to those points. (Are the endpoints included in the solution set?)
Write your answer with interval notation:

d. Use the graph to solve $|x-3|>5$
Write your answer with interval notation:

2. Solving with a number line

a. Translate the equation $|x-3|<5$ into a sentence about distance.

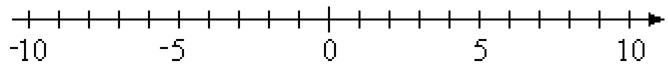
b. Locate 3 on the number line below, and shade the points closer than 5 units from 3.



c. Write your answer with interval notation:

d. Now use the number line to solve

$$|x-3| > 5$$



(Shade the points farther than 5 units from 3.)

Write your answer with interval notation:

3. Solving with algebra

a. First, solve the equation $|x-3| = 5$. You should get two solutions, one on either side of 3. These are the boundary points.

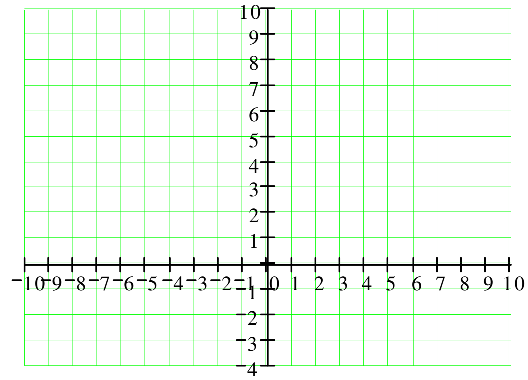
b. Now consider the inequality $|x-3| < 5$. This absolute value inequality is equivalent to the compound inequality: $-5 < x-3 < 5$
Solve the inequality and verify that you get the same solution as in part **2(c)** above.

c. Now we'll solve $|x-3| > 5$. This absolute value inequality is equivalent to the compound inequality: $x-3 > 5$ or $x-3 < -5$
Solve the inequality and verify that you get the same solution as in part **2(d)** above.

Activity 3 More Inequalities

1. Solve the inequality $|2x-5| \le 6$ in two ways:

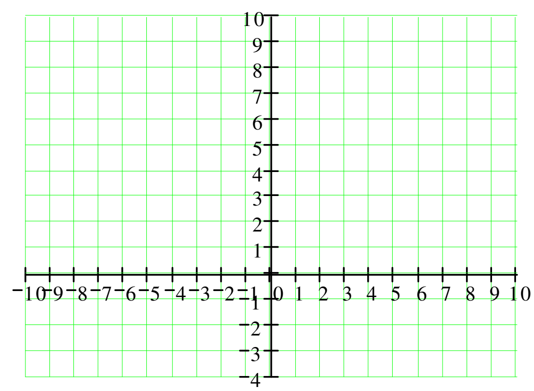
a. Graph $y = |2x-5|$ on the grid, and use the graph to solve the inequality.
Write your answer in interval notation:



b. Solve algebraically: $|2x-5| \le 6$ is equivalent to: $-6 \le 2x-5 \le 6$

2. Solve the inequality $|3x-6| \ge 3$ in two ways:

a. Graph $y = |3x-6|$ on the grid, and use the graph to solve the inequality.
Write your answer in interval notation:



b. Solve algebraically: $|3x-6| \ge 3$ is equivalent to:
$3x-6 \ge 3$ or $3x-6 \le -3$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Sketch a graph whose shape models a situation
- Choose one of the basic graphs to fit a situation or a set of data
- Decide whether the graph of a function is increasing or decreasing, concave up or concave down
- Use absolute value notation to write statements about distance
- Use graphs to solve absolute value equations and inequalities
- Solve absolute value equations and inequalities algebraically
- Express error tolerances using absolute value notation

Check Your Understanding

1. Which basic function is increasing but concave down? Which basic function is decreasing but concave up?

2. How do we write "the distance between x and 5" in mathematical notation?

3. How many solutions are there to the equation $|2x-7|=k$, if $k>0$?

4. Do you prefer to solve absolute value inequalities algebraically, or with a graph? Why?

Review Lesson: Laws of Exponents

I First Law: Product of Powers

1. Write out the powers in each expression to see whether the statement is **true or false**. You can also use your calculator to help you decide.

a. $4^2 \cdot 4^3 = 16^6$ $(4 \cdot 4)\,(4 \cdot 4 \cdot 4) \stackrel{?}{=} 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$

b. $4^2 \cdot 4^3 = 4^5$

c. $2^3 \cdot 3^2 = 6^5$

d. $5^3 \cdot 5^3 = 2 \cdot 5^3$

e. $4^2 \cdot 3^2 = 12^2$

f. $6^2 + 6^3 = 6^5$

Conclusion: We can simplify a **product of powers** if they have the same __________ . We _______________ the exponents, and _______________________ base.

2. Substitute values for the variables and evaluate to decide whether each statement is **true or false**. (Choose different values for x and y, but do not choose 0 or 1.)

a. $x^2 \cdot x^3 = x^6$

b. $x^2 \cdot x^3 = x^5$

c. $x^3 \cdot y^2 = (xy)^5$

d. $x^3 \cdot x^3 = 2x^3$

e. $x^2 \cdot y^2 = (xy)^2$

f. $x^2 + x^3 = x^5$

3. Simplify each expression. If it cannot be simplified, write "c. b. s."

a. $a^4 \cdot a^3$ **b.** $a^4 + a^3$

c. $a^4 \cdot b^2$ **d.** $a^4 \cdot b^4$

e. $a^6 \cdot a^6$ **f.** $a^6 + a^6$

II Second Law: Quotient of Powers

1. Write out the powers in each expression to see whether the statement is **true or false**. You can also use your calculator to help you decide.

a. $\frac{3^8}{3^2} = 3^4$ $\qquad \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} \stackrel{?}{=} 3 \cdot 3 \cdot 3 \cdot 3$

b. $\frac{5^9}{5^3} = 5^6$

c. $\frac{6^5}{2^3} = 3^2$

d. $\frac{6^5}{2^5} = 3^5$

e. $\frac{4^2}{4^5} = \frac{1}{4^3}$

f. $\frac{3^4}{3^{12}} = \frac{1}{3^3}$

Conclusion: We can simplify a **quotient of powers** if they have the same __________ . We ___________ the exponents, and ______________________ base.

2. Substitute values for the variables and evaluate to decide whether each statement is **true or false**. (Choose different values for x and y, but do not choose 0 or 1.)

a. $\frac{x^8}{x^2} = x^4$

b. $\frac{x^9}{x^3} = x^6$

c. $\frac{x^5}{y^3} = \left(\frac{x}{y}\right)^2$

d. $\frac{x^5}{y^5} = \left(\frac{x}{y}\right)^5$

e. $\frac{x^2}{x^5} = \frac{1}{x^3}$

f. $\frac{x^4}{x^{12}} = \frac{1}{x^3}$

3. Simplify each expression. If it cannot be simplified, write "c. b. s."

a. $\dfrac{a^9}{a^3}$ **b.** $\dfrac{b^7}{b^4}$

c. $\dfrac{a^6}{b^2}$ **d.** $\dfrac{a^6}{b^6}$

e. $\dfrac{m^4}{m^8}$ **f.** $m^8 - m^4$

III Third Law: Raising a Power to a Power

1. Write out the powers in each expression to see whether the statement is **true or false**. You can also use your calculator to help you decide.

a. $(2^3)^4 = 2^7$ $(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2) \stackrel{?}{=} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

b. $(2^3)^4 = 2^3 \cdot 2^4$

c. $(3^2)^5 = 3^{10}$

d. $(3^2)^5 = (3^5)^2$

e. $(5^2)^3 = 25^3$

f. $(5^2)^3 = 5^8$

Conclusion: We can simplify a **power of a power**. We ___________ the exponents, and ______________________ base.

2. Substitute values for the variables and evaluate to decide whether each statement is **true or false**. (Choose different values for x and y, but do not choose 0 or 1.)

a. $(x^3)^4 = x^7$ **b.** $(x^3)^4 = x^3 \cdot x^4$

c. $(y^3)^2 = y^9$ **d.** $(y^5)^2 = (y^2)^5$

3. Simplify each expression. If it cannot be simplified, write "c. b. s."

a. $(n^3)^3$ **b.** $(b^5)^4$

c. $(x^{10})^{10}$ **d.** $(x^{20})^{25}$

IV Fourth Law: Raising a Product to a Power

1. Write out the powers in each expression to see whether the statement is **true or false**. You can also use your calculator to help you decide.

a. $(5 \cdot 3)^3 = 5 \cdot 3^3$ $\quad (5 \cdot 3)(5 \cdot 3)(5 \cdot 3) \stackrel{?}{=} 5 \cdot 3 \cdot 3 \cdot 3$

b. $(5 \cdot 3)^3 = 5^3 \cdot 3^3$

c. $(2 + 3)^4 = 2^4 + 3^4$

d. $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

e. $(1.5 \times 10^3)^4 = 1.5^4 \times 10^{12}$

f. $(100 - 25)^3 = 100^3 - 25^3$

Conclusion: We can simplify a **power of a product** by raising each __________ to the exponent. We cannot simplify a power of a ___________ .

2. Substitute values for the variables and evaluate to decide whether each statement is **true or false**. (Choose different values for x and y, but do not choose 0 or 1.)

a. $(xy^3)^2 = xy^6$ **b.** $(x^3y)^3 = x^9y^3$

c. $(2x^4)^2 = 2x^8$ **d.** $(3x^3)^4 = 81x^{12}$

e. $(x^2y^2)^3 = x^6y^6$ **f.** $(x^2 + y^2)^3 = x^6 + y^6$

3. Simplify each expression. If it cannot be simplified, write "c. b. s."

a. $(5p^2)^5$ **b.** $(5 + p^2)^5$

c. $(a^3 - b^4)^4$ **d.** $(wz^2)^6$

e. $(x^{12}y^6)^{12}$ **f.** $(1 + 2z^{10})^6$

V Fifth Law: Raising a Quotient to a Power

1. Write out the powers in each expression to see whether the statement is **true or false**. You can also use your calculator to help you decide.

a. $\left(\frac{8}{2}\right)^2 = \frac{8^2}{2^2}$ $\qquad \left(\frac{8}{2}\right)\left(\frac{8}{2}\right) \stackrel{?}{=} \frac{8 \cdot 8}{2 \cdot 2}$

b. $\left(\frac{9}{10}\right)^3 = \frac{9^3}{10^3}$

c. $\left(\frac{1}{4}\right)^5 = \frac{5}{4^5}$

d. $\left(\frac{2}{5}\right)^{10} = \frac{20}{50}$

> **Conclusion:** We can simplify a **power of a quotient** by raising both the __________ and _______________ to the exponent.

2. Substitute values for the variables and evaluate to decide whether each statement is **true or false**. (Choose different values for x and y, but do not choose 0 or 1.)

a. $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$

b. $\left(\frac{x}{x+y}\right)^3 = \frac{1}{y^3}$

c. $\left(\frac{5x}{y^4}\right)^4 = \frac{5x^4}{y^{16}}$

d. $\left(\frac{3+x^2}{2-y^2}\right)^3 = \frac{3^3+x^6}{2^3-y^6}$

Lesson 23 Integer Exponents

Activity 1 Power Functions

1. State the values of k and p for the two examples below:

a. $V = f(r) = \frac{4}{3}\pi\, r^3$, the volume of a sphere as a function of its radius

$k =$ $p =$

b. $L = f(T) = 0.8125\, T^2$, the length of a pendulum as a function of its period

$k =$ $p =$

2. Write each expression as a power function using negative exponents. State the values of k and p.

a. $h(s) = \dfrac{9}{s^3}$

b. $f(v) = \dfrac{3}{8v^6}$

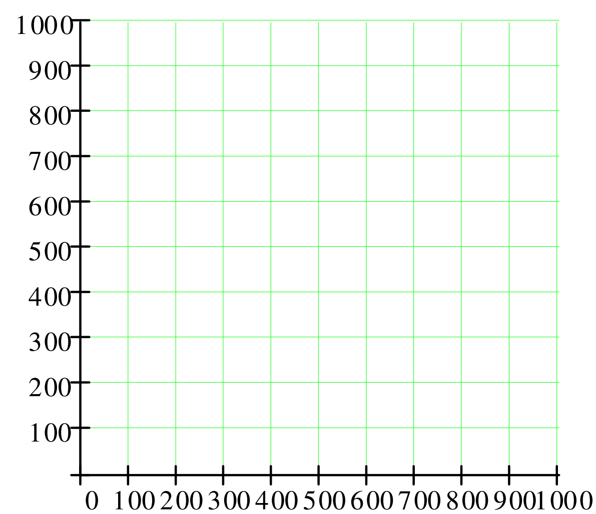
c. $g(t) = \dfrac{1}{(5t)^4}$

3. The Trebuchet

In the middle ages in Europe, castles were used as defensive strongholds. An attacking force would build a huge catapult called a **trebuchet** to hurl rocks and scrap metal inside the castle walls. The engineers could adjust its range by varying the mass (or weight) of the missiles. The mass, m, of the projectile is inversely proportional to the square of the distance, d, to the target.

a. Use a negative exponent to write a formula for m as a function of d, $m = f(d)$.

b. The engineers test the trebuchet with a 30-pound lead ball, which lands 948 yards away. Find the constant of proportionality, and rewrite your formula for m. (Round your value of k to the nearest million.)



c. Graph $m = f(d)$.

d. The trebuchet is 500 yards from the courtyard within the castle. What size missile will hit the target?

e. The largest missile the trebuchet can handle is 300 pounds. If the attackers would like to hurl a 300-pound rock at the castle, how close must they bring their trebuchet?

4. Radar

Airplanes use radar to detect the distances to other objects. A radar unit transmits a pulse of energy which bounces off a distant object, and the echo of the pulse returns to the sender. The power, P, of the returning echo is inversely proportional to the fourth power of the distance, d, to the object.

a. A radar operator receives an echo of 5×10^{-10} watt from an aircraft 2 nautical miles away. Express the power of the echo received in **picowatts**. (1 picowatt $=$ 10^{-12} watt.)

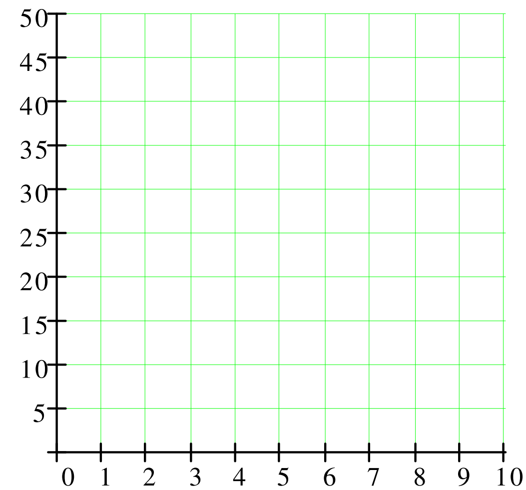
b. Write a function that expresses P in terms of d using negative exponents. Use picowatts for the units of power.

c. Complete the table of values for the power of the echo received from objects at various distances.

d (nautical miles)	4	5	7	10
P (picowatts)				

d. Sketch a graph of P as a function of d. Use units of picowatts on the vertical axis.

e. Radar units can typically detect signals as low as 10^{-13} watt. How far away is an aircraft whose echo is 10^{-13} watt? (Hint: Convert 10^{-13} watt to picowatts.)



Activity 2 Using the Laws of Exponents

1. Simplify if possible using the first law.

a. $x^6 \cdot x^{-3}$

b. $x^{-6} \cdot x^3$

c. $x^{-6} \cdot x^{-3}$

d. $x^3 + x^{-6}$

e. $x^{-3} - x^{-6}$

f. $x^3 + x^{-3}$

2. Simplify if possible using the second law.

a. $\dfrac{x^6}{x^3}$

b. $\dfrac{x^{-6}}{x^3}$

c. $\dfrac{x^6}{x^{-3}}$

d. $\dfrac{x^{-6}}{x^{-3}}$

e. $\dfrac{x^3}{x^{-6}}$

f. $\dfrac{x^{-3}}{x^{-6}}$

3. Simplify if possible using the third law.

a. $(x^6)^{-3}$

b. $(x^{-6})^3$

c. $(x^{-6})^{-3}$

d. $(-x)^{-3}$

e. $(-x)^{-6}$

f. $(-x^3)^{-6}$

4. Simplify if possible using the fourth and fifth laws.

a. $(2x)^{-3}$

b. $\left(\dfrac{2}{x}\right)^{-3}$

c. $(2x^4)^{-3}$

d. $(2x^{-4})^3$

e. $(2x^{-4})^{-3}$

f. $(-2x^{-4})^{-3}$

g. $\left(\dfrac{2}{x^{-4}}\right)^{3}$

h. $\left(\dfrac{2}{x^{-4}}\right)^{-3}$

5. Which of the following statements are true for **all** non-zero values of a and b? (You may want to try out some specific values of a and b before you decide!)

a. $\dfrac{1}{a}+\dfrac{1}{b} \stackrel{?}{=} \dfrac{1}{a+b}$

b. $a^{-1}+b^{-1} \stackrel{?}{=} (a+b)^{-1}$

c. $a^2+b^2 \stackrel{?}{=} (a+b)^2$

d. $a^{-2}+b^{-2} \stackrel{?}{=} (a+b)^{-2}$

e. $\dfrac{1}{a^{-1}}+\dfrac{1}{b^{-1}} \stackrel{?}{=} a+b$

f. $\dfrac{1}{a^{-1}+b^{-1}} \stackrel{?}{=} a+b$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Evaluate negative exponents
- Write a formula for a power function
- Graph and analyze power functions
- Solve equations involving negative exponents
- Simplify expressions using the laws of exponents

Check Your Understanding

1. Explain the difference between each pair of expressions.
a. -2^3 and 2^{-3} **b.** $-x^4$ and x^{-4} **c.** -2^n and 2^{-n}

2. Write a power function for "y varies inversely with the cube of x."

3. How do you convert a value in watts to picowatts?

4. Explain how to decide which law of exponents to use when simplifying an expression.

Lesson 24 Roots and Radicals

Activity 1 Maximum Speed for Ships

When a ship moves through the water, it creates waves that impede its own progress. Because of this resistance, there is an upper limit to the speed at which a ship can travel. This maximum speed, or hull velocity, is given in knots by

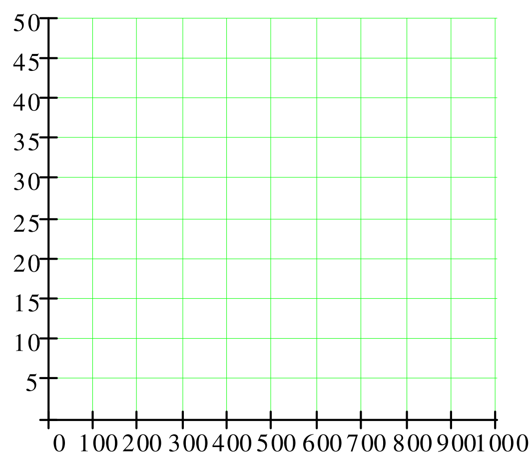
$$v_{\text{max}} = f(L) = 1.3\,L^{1/2}$$

where L is the length of the vessel in feet.

a. Complete the table of values for v_{max} as a function of L.

L (feet)	0	200	400	600	800	1000
v_{max} (knots)						
v_{c} (knots)						

b. Graph maximum speed against vessel length.



c. Write and solve an equation to answer the question: What is the length of a vessel whose top speed is 20 knots?

d. The world's largest ship, the oil tanker Jahre Viking, is 1054 feet long. What is its top speed?

e. Write the formula for v_{max} using a decimal fraction for the exponent.

Write the formula for v_{max} using a radical.

f. As a ship approaches its maximum speed, the power required increases sharply. Therefore, most merchant ships are designed to cruise at speeds no higher than $v_{\text{c}} = 0.8\,L^{1/2}$. Graph the cruising speed v_{c} on the same axes with v_{max}.

g. What is the cruising speed of the Jahre Viking? What percent of its maximum speed is that?

Activity 2 Heart Rate

An animal's heart rate is related to its size or mass, with smaller animals generally having faster heart rates. The heart rates of mammals are given approximately by the power function

$$H = f(m) = km^{-1/4}$$

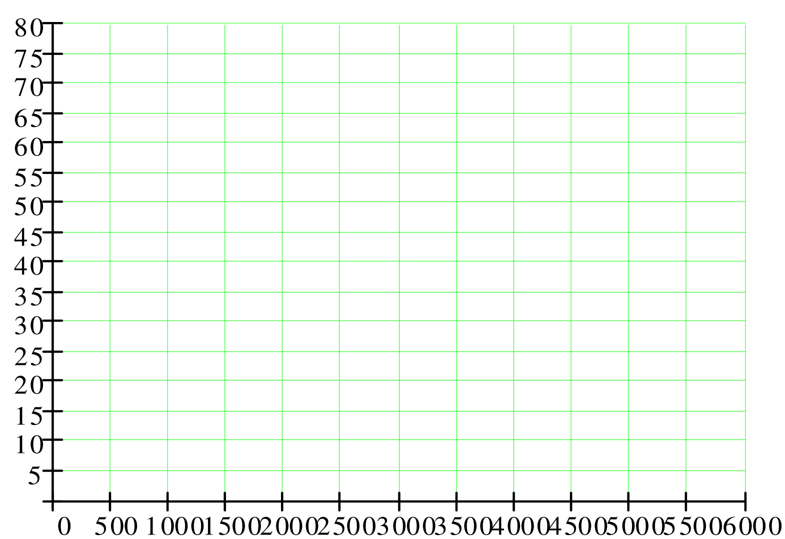
where m is the animal's mass and k is a constant.

a. A typical human male weighs about 70 kilograms and has a resting heart rate of 70 beats per minute. Find the constant of proportionality, k, and write a formula for H as a function of m.

b. Complete the table with the heart rates of the mammals whose masses are given

Animal	Cat	Wolf	Horse	Polar Bear	Elephant
Mass (kg)	4	80	300	800	5400
Heart Rate					

c. Sketch a graph of H for masses up to 6000 kilograms.



d. What would be the mass of a mammal whose heart rate is 40 beats per minute?

e. Write the formula for H using a decimal fraction for the exponent.

Write the formula for H using a radical.

Activity 3 Laws of Exponents

1. Complete the box below.

Laws of Exponents
I. $\boldsymbol{a^m \cdot a^n =}$
II. $\boldsymbol{\left(\dfrac{a^m}{a^n}\right) =}$
III. $\boldsymbol{(a^m)^n =}$
IV. $\boldsymbol{(a\,b)^n =}$
V. $\boldsymbol{\left(\dfrac{a}{b}\right)^n =}$

2. a. Use exponents to explain why $\sqrt{\sqrt{x}} = \sqrt[4]{x}$.

b. Use exponents to explain why $\dfrac{\sqrt{x}}{\sqrt[4]{x}} = \sqrt[4]{x}$.

c. Use exponents to explain why $\sqrt[3]{x}\sqrt[6]{x} = \sqrt{x}$.

3. Explain in words how to evaluate each expression. Then illustrate with $x = 9$.

a. $-x^2$

b. x^{-2}

c. $\frac{1}{2}x$

d. $x^{1/2}$

e. $-x^{1/2}$

f. $x^{-1/2}$

4. Which of the following statements are true for **all** positive values of the variables?

a. $(a^2+b^2)^{1/2}=a+b$

b. $(x^2y^2)^{1/2}=xy$

c. $(c^{1/2}-d^{1/2})^2=c-d$

d. $(u^{1/2}v^{1/2})^2=uv$

e. $\sqrt[3]{n^3-p^3}=n-p$

f. $\sqrt[3]{s^3t^3}=st$

g. $\left(\sqrt[3]{m}+\sqrt[3]{h}\right)^3=m+h$

h. $\left(\sqrt[3]{w}\sqrt[3]{z}\right)^3=wz$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Evaluate fractional powers and roots
- Convert between radical and exponential notation
- Graph and analyze power functions
- Solve equations involving radicals
- Simplify expressions using the laws of exponents

Check Your Understanding

1. Explain why $x^{1/4}$ is a reasonable notation for $\sqrt[4]{x}$.

2. What does the notation $a^{0.125}$ mean? How is it evaluated?

3. Express each of the following algebraic notations in words, and evaluate each for $x=16$:

$$4x,\ x^4,\ \frac{x}{4},\ \frac{1}{4}x,\ x^{1/4},\ x-4,\ x^{-4},\ x^{-1/4}$$

4. In Activity 2, explain why the graph of $H(m)=km^{-1/4}$ is decreasing.

Lesson 25 Rational Exponents

Activity 1 Kepler's Law

Kepler's law gives a relation between the period p of a planet's revolution and its average distance a from the sun:

$$p^2 = Ka^3$$

where $K = 1.243 \times 10^{-6}$, a is in millions of miles, and p is in years.

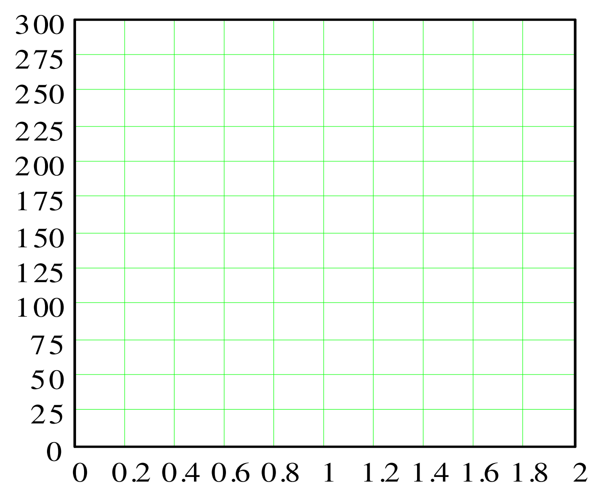
a. Solve Kepler's law for a in terms of p. Write your equation in two ways: using a fractional exponent, and using a radical.

b. Complete the table for the four inner planets. Give the values of a in millions of miles.

Planet	Mercury	Venus	Earth	Mars
p (years)	0.241	0.615	1.000	1.881
a (10^6 miles)				

c. Graph a as a function of p.

d. The planet Neptune is 4.497×10^3 million miles from the sun. Find its period of revolution.



Activity 2 Vampire Bats

Small animals like bats cannot survive for long without eating. The weight W of a typical vampire bat decreases over time until its next meal, according to the function

$$W = f(h) = 130.25h^{-0.126}$$

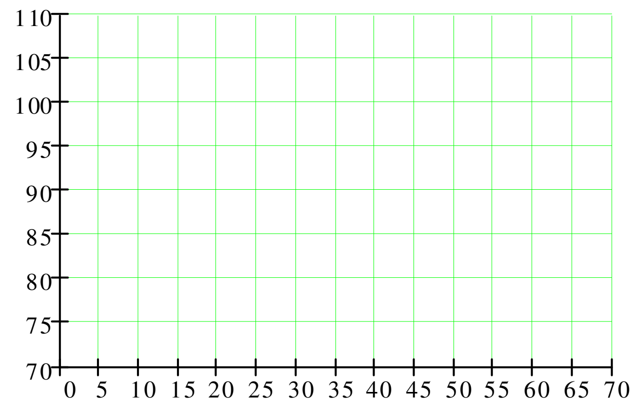
where h is the number of hours since the bat's most recent meal.

a. Complete the table showing the bat's weight after fasting for h hours.

h (hours)	5	10	20	40	50	70
W (grams)						

b. Graph the equation.

c. A bat can survive for 60 hours without food. What is the bat's weight at the point of starvation? Locate this point on the graph.



d. When the bat's weight has dropped to 90 grams, how long has it been without food?

How long can it survive before its next meal?

e. Complete the table with the number of hours since eating at various weights, and label the corresponding points on the graph. Round to the nearest hour.

Weight (grams)	97.5	92.5	85	80
Hours since Eating				
Point on Graph	A	B	C	D

f. Compute the slope, including units, from point A to point B, and from point C to point D.

$m_{AB} =$

$m_{CD} =$

g. Vampire bats sometimes donate blood (through regurgitation) to other bats that are close to starvation. Suppose a bat at point A on the curve donates 5 grams of blood to a bat at point D. How many hours of life does the recipient gain from the 5 grams of food? How many hours of life does the donor give up?

Activity 3 Laws of Exponents

1. Simplify by using the laws of exponents. Write your answers as radicals.

a. $x^{3/4}x^{5/4}$

b. $\left(x^{3/2}\right)^{5/2}$

c. $\dfrac{z^{1/3}}{z^{2/3}}$

d. $\dfrac{w^{1/5}w^{-3/5}}{w^{-4/5}}$

2. Simplify by using the laws of exponents. Write your answers as radicals.

a. $b^{1/3}b^{2/5}$

b. $\dfrac{a^{5/4}}{a^{-4/3}}$

c. $\left(-3\,c^{4/3}\right)^{-2}$

d. $\dfrac{\left(a^{3/2}b^{-1/4}\right)^{1/3}}{a^{3/8}}$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Evaluate powers with rational exponents
- Convert between radical and exponential notation
- Graph and analyze power functions with rational exponents
- Solve equations involving rational exponents
- Simplify expressions using the laws of exponents

Check Your Understanding

1. What does the notation $a^{0.6}$ mean?

2. Explain how to evaluate the function $f(x) = x^{-3/4}$ for $x = 625$, without using a calculator.

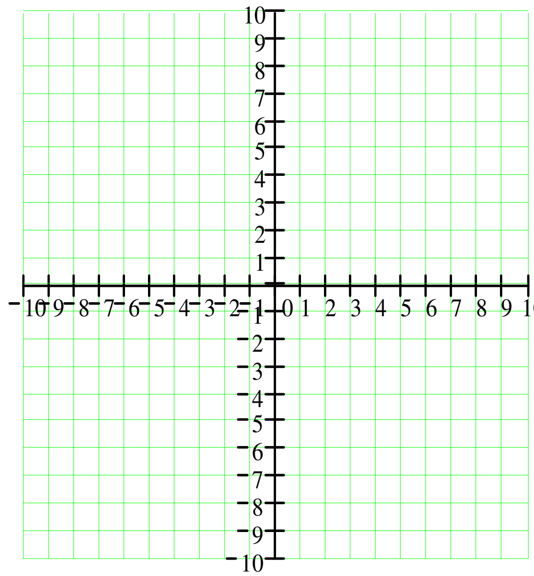
3. Explain why $x\sqrt{x} = x^{1.5}$.

4. Suppose $M = 9 \times 10^{14}$. Explain why $\sqrt{M} = 3 \times 10^{7}$.

Lesson 26 Distance and Midpoint Formulas

Activity 1 The Distance Formula

1. a. Plot the points $(-5, 3)$ and $(3, -9)$, and draw the line segment joining them.



b. Draw horizontal and vertical line segments to make a right triangle. What are the coordinates at the vertex of the right angle?

c. Find the length of each leg of the right triangle. We'll call those lengths a and b.

$a =$

$b =$

(Do you see how to calculate a and b from the coordinates of the given points?)

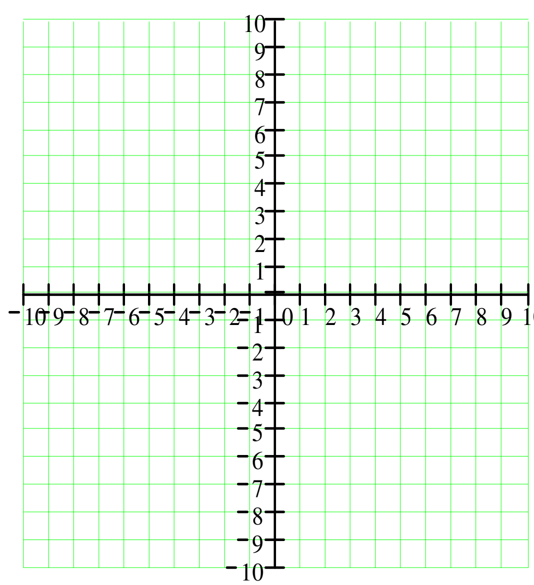
$a =$

$b =$

d. Use the Pythagorean theorem to calculate the **distance** between $(-5, 3)$ and $(3, -9)$.

2. Now we'll do the same thing in general:

a. Plot two points $(x_1,\ y_1)$ and $(x_2,\ y_2)$, and draw the line segment joining them.



b. Draw horizontal and vertical line segments to make a right triangle. What are the coordinates at the vertex of the right angle?

c. Find the length of each leg of the right triangle. We'll call those lengths a and b.

$a =$

$b =$

d. Use the Pythagorean theorem to calculate the **distance** between $(x_1,\ y_1)$ and $(x_2,\ y_2)$.

3. a. In part (2), you derived the **distance formula**.

> The **distance** d between points $P_1(x_1,\, y_1)$ and $P_2(x_2,\, y_2)$ is
>
> $d =$

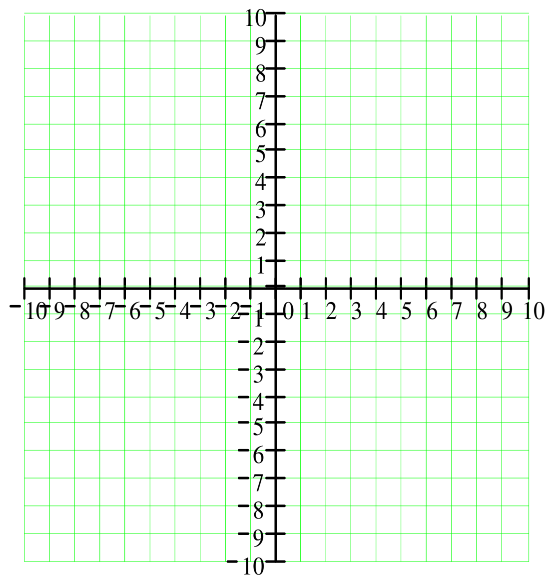
b. Use the formula to find the distance between $(-5, 2)$ and $(7, -3)$.

Activity 2 The Midpoint Formula

1. a. Plot the points $(-5, 3)$ and $(3, -9)$, and draw the line segment joining them.

b. Draw horizontal and vertical line segments to make a right triangle.

c. Now start at $(-5,\, 3)$ and move half the horizontal distance and half the vertical distance to $(3,\, -9)$. What are the coordinates of the new point?



d. The new point is called the **midpoint** of the segment joining $(-5, 3)$ and $(3, -9)$. It is half-way between $(-5, 3)$ and $(3, -9)$.

2. How can we find a formula for calculating the coordinates of the midpoint?

a. When we calculate the **average** of two numbers, we find a number that is half-way between them. For example, if your test scores are 70 and 90, your test average is

$$\text{average} = \frac{70 + 90}{2} = \frac{160}{2} = 80$$

b. For the two points in part (1), calculate the average of their x-coordinates and the average of their y-coordinates.

$$\overline{x} = \frac{x_1 + x_2}{2} =$$
$$\overline{y} = \frac{y_1 + y_2}{2} =$$

You should get the same point you found in part (1).

3. a. We now have a formula for the midpoint.

> The **midpoint** of the line segment joining the points $P_1(x_1,\, y_1)$ and $P_2(x_2, y_2)$ is the point $M(\overline{x}, \overline{y})$, where
>
> $\boldsymbol{\overline{x}} =$ and $\boldsymbol{\overline{y}} =$

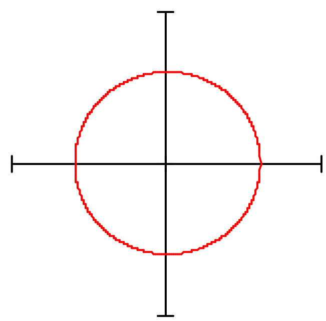
b. Use the formula to find the midpoint of the line segment joining $(-5, 2)$ and $(7, -3)$.

Activity 3 Circles

1. a. A **circle** is the set of all points in a plane that lie at a given distance, called the **radius**, from a fixed point called the **center.** We can use the distance formula to find an equation for a circle.

b. The center of the circle shown is the origin, $(0, 0)$. The distance from the origin to any point $P(x, y)$ on the circle is r. Therefore,

$$\sqrt{(x-0)^2 + (y-0)^2} = r$$



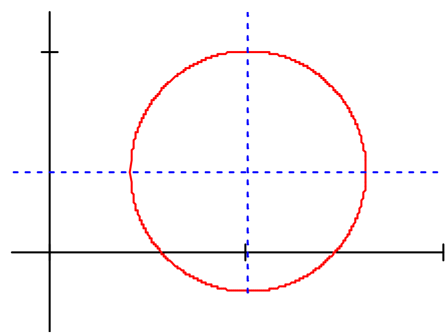
or, squaring both sides,

$$(x-0)^2 + (y-0)^2 = r^2$$

Thus, the equation for a circle of radius r centered at the origin is

$$\boldsymbol{x^2 + y^2 = r^2}$$

c. Use the distance formula to write a formula for the circle whose radius is r and whose center is (h, k).



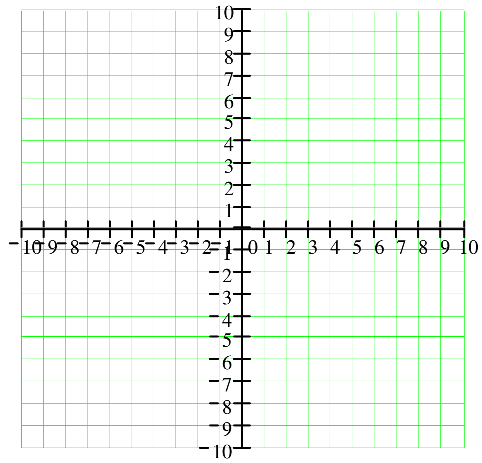
2. a. Square both sides to obtain a formula for a circle.

> The **standard form** for the equation for a **circle** of radius r centered at the point $(h,\, k)$ is

b. Write an equation for the circle with center at $(4, -3)$ with radius $2\sqrt{6}$.

3. a. State the center and radius of the circle

$(x+3)^2 + (y+2)^2 = 16$



b. Graph the circle on the grid.

c. Expand the left side of the equation in part (a), and write it in **general quadratic form**:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

4. a. The **standard form** for a circle is useful because we can see the center and the radius right in the equation, so we can graph the circle. But often we get an equation like the one in part (3c). Then we have to convert the equation to standard form by **completing the square**.

b. Follow the steps to complete the square:

$$x^2 + y^2 - 14x + 4y = -25$$

Collect the x-terms together
and the y-terms together

Complete the square for x and y

Write each expression as a perfect
square; simplify the right side

The circle has center __________ and radius _____________.

c. Write the equation for the circle in standard form:

$$x^2 + y^2 - 10x - 6y - 24 = 0$$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Find the distance between two points
- Find the midpoint of a line segment
- Graph a circle
- Find the equation for a circle
- Use completing the square to write the equation of a circle in standard form

Check Your Understanding

1. State the Pythagorean theorem, then solve for c. Compare this equation with the distance formula; how are they similar?

2. In the midpoint formula, why do we add the coordinates of the two points, instead of subtracting them, as we do in the distance formula?

3. What fact about circles does the equation of a circle state?

4. What algebraic technique do we use to write the equation of a circle in standard form?

Lesson 27 Working with Radicals A

Activity 1 Equivalent Radicals

We use the **product and quotient rules for radicals** to **simplify radicals**. To "simplify" a radical does **not** mean to find a decimal approximation using a calculator!! The new expression is an **exact equivalent** for the original radical, not an approximation.

1. Use your calculator to verify each calculation.

a. $\sqrt{12} = \sqrt{\mathbf{4} \cdot 3} = \sqrt{\mathbf{4}}\sqrt{3} = \mathbf{2}\sqrt{3}$ (Note that $2\sqrt{3}$ means 2 times $\sqrt{3}$.)

b. $\sqrt{45} = \sqrt{\mathbf{9} \cdot 5} = \sqrt{\mathbf{9}}\sqrt{5} = \mathbf{3}\sqrt{5}$

c. $\sqrt[3]{40} = \sqrt[3]{\mathbf{8} \cdot 5} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{5} = \mathbf{2}\sqrt[3]{5}$

d. $\sqrt[4]{162} = \sqrt[4]{\mathbf{81} \cdot 2} = \sqrt[4]{\mathbf{81}} \cdot \sqrt[4]{2} = \mathbf{3}\sqrt[4]{2}$

2. There is a difference between simplifying a radical and finding a decimal approximation for a radical! Compare:

Simplify $\sqrt{45}$ Answer: $\sqrt{45} = \sqrt{\mathbf{9}}\sqrt{5} = \mathbf{3}\sqrt{5}$ (no calculators!)

Approximate $\sqrt{45}$ Answer: $\sqrt{45} \approx 6.708$ (by calculator)

3. Now you simplify some. Look for a **perfect square** that divides evenly into the radicand:

a. $\sqrt{200} =$

b. $\sqrt{48} =$

Look for a **perfect cube** that divides evenly into the radicand:

c. $\sqrt[3]{72} =$

d. $\sqrt[3]{250} =$

Activity 2 Variables under the Radical

1. We can do this with variables, too. For example:

$\sqrt{x^4} = x^2$ because $(x^2)^2 = x^4$

$\sqrt{x^{10}} = x^5$ because $(x^5)^2 = x^{10}$

a. $\sqrt{x^6} =$ ________ because $(\_\_\_\_)^2 = x^6$

b. $\sqrt[3]{x^6} =$ ________ because $(x^2)^3 = x^6$

c. $\sqrt[3]{x^{15}} =$ ________ because $(\_\_\_\_)^3 = x^{15}$

d. $\sqrt[4]{x^{12}} =$ ________ because $(\_\_\_\_)^4 = x^{12}$

2. Now we'll simplify some roots of products. Finish each example:

a. $\sqrt{81\,x^8} = \sqrt{81}\sqrt{x^8} =$

b. $\sqrt{16\,x^{16}} = \sqrt{16}\sqrt{x^{16}} =$

c. $\sqrt[3]{8x^9} = \sqrt[3]{8}\sqrt[3]{x^9} =$

d. $\sqrt[4]{81\,x^{20}} =$

3. Here are some roots of quotients. Finish each example:

a. $\sqrt{\dfrac{9a^8}{16b^4}} = \dfrac{\sqrt{9a^8}}{\sqrt{16b^4}} =$

b. $\sqrt[3]{\dfrac{-27y^9}{125z^6}} =$

Activity 3 Simplifying Radicals

1. What about simplifying the square root of an odd power? An even power is a perfect square, so we factor off one power of x. For example,

$$x^7 = \boldsymbol{x^6} \cdot x \qquad \text{and} \qquad x^{13} = \boldsymbol{x^{12}} \cdot x$$

a. $\sqrt{x^7} = \sqrt{x^6}\sqrt{x} = x^3\sqrt{x}$ (Note that $x^3\sqrt{x}$ means x^3 times $\sqrt{x}$.)

b. $\sqrt{x^{13}} = \sqrt{x^{12}}\sqrt{x} =$

For cube roots, we factor out a power whose exponent is divisible by three. For example,

$$x^8 = \boldsymbol{x^6} \cdot x^2 \qquad \text{and} \qquad x^{13} = \boldsymbol{x^{12}} \cdot x$$

c. $\sqrt[3]{x^8} = \sqrt[3]{x^6} \cdot \sqrt[3]{x^2} = x^2\ \sqrt[3]{x^2}$ (We cannot simplify $\sqrt[3]{x^2}$ any further.)

d. $\sqrt[3]{x^{13}} = \sqrt[3]{x^{12}} \cdot \sqrt[3]{x} =$

2. Now we can simplify any radical. To simplify a square root, factor out any perfect squares from the radicand. Factor the numbers and each variable separately.

a. $\sqrt{12t^5} = \sqrt{4t^4 \cdot 3t} = \sqrt{4t^4}\sqrt{3t} =$ ______ $\cdot \sqrt{3t}$

b. $\sqrt{27w^9} = \sqrt{9w^8 \cdot 3w} = \sqrt{9w^8}\sqrt{3w} =$ _____________

c. $\sqrt{72b^{16}c^{15}} =$

To simplify a cube root, factor out any perfect cubes from the radicand.

d. $\sqrt[3]{32m^{11}} = \sqrt[3]{8m^9}\ \sqrt[3]{4m^2} =$ ______ $\cdot \sqrt[3]{4m^2}$

e. $\sqrt[3]{81a^{12}b^8} = \sqrt[3]{27a^{12}b^6}\ \sqrt[3]{3b^2} =$ _______________

f. $\sqrt[3]{40x^{14}y^9} =$

3. Now you try some. Simplify:

a. $\sqrt{18xy^3} =$

b. $\sqrt{80\,a^5b^4} =$

c. $\sqrt[3]{24\,m^3p^2} =$

d. $\sqrt[3]{64\,w^4z^2} =$

Lesson 27 Working with Radicals B

Activity 1 Products and Quotients of Radicals

1. We can combine the radicands in products or quotients of radicals, as long as both radicals have the same **index**. For example:

a. $\sqrt{12x}\sqrt{3x} = \sqrt{(12x)(3x)} = \sqrt{36x^2} = 6x$

b. $\sqrt[4]{2m^3}\sqrt[4]{8m} = \sqrt[4]{(2m^3)(8m)} = \sqrt[4]{\quad ? \quad} =$ ____________

2. Complete the following examples of quotients:

a. $\dfrac{\sqrt{18xy^3}}{\sqrt{2y}} = \sqrt{\dfrac{18xy^3}{2y}} =$

b. $\dfrac{\sqrt[3]{200a^8}}{\sqrt[3]{5a^2}} = \sqrt[3]{\dfrac{200a^8}{5a^2}} =$

Activity 2 Sums and Differences of Radicals

1. Recall that we can combine radicals by multiplying and dividing, but not by adding and subtracting. For example,

$$\sqrt{12}\sqrt{3} = \sqrt{36} \qquad \text{and} \qquad \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4}$$

$$\textbf{but} \quad \sqrt{12} + \sqrt{3} \neq \sqrt{15} \qquad \text{and} \qquad \sqrt{12} - \sqrt{3} \neq \sqrt{9}$$

(You should check these on your calculator.) Decide whether each of the following is true or false (use your calculator):

a. $\sqrt{7} + \sqrt{7} = \sqrt{14}$?

b. $\sqrt{7} + \sqrt{7} = 2\sqrt{7}$?

c. $\sqrt{7} - \sqrt{5} = \sqrt{2}$?

d. $6\sqrt{5} - 2\sqrt{5} = 4\sqrt{5}$?

2. We can only add or subtract **like radicals**, that is, radicals with the same index and the same radicand. This is similar to adding or subtracting like terms. Compare:

$3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$ and $3x + 2x = 5x$

$8\sqrt{5} - 3\sqrt{2}$ c.b.s. and $8x - 3y$ c.b.s.

$4\sqrt{x} + 3\sqrt{xy}$ c.b.s. and $4x + 3xy$ c.b.s.

$\sqrt{5} - \sqrt{5x}$ c.b.s. and $5 - 5x$ c.b.s.

Notice that the radicand does not change when we add or subtract like radicals, only the coefficients are combined. (The same is true when we add or subtract like terms.)

True or False:

a. $a\sqrt{5} + a\sqrt{2} = a\sqrt{7}$?

b. $\sqrt{11} + \sqrt[3]{11} = \sqrt[5]{11}$?

c. $2x\sqrt{3x} + 5\sqrt{3x} = 7x\sqrt{6x}$?

d. $\sqrt[3]{25} - \sqrt[3]{15} = \sqrt[3]{10}$?

3. Now you try some. Simplify by combining like terms:

a. $5\sqrt{6} - 2\sqrt{6} + 6\sqrt{2} =$

b. $8\sqrt[3]{a^2} - 5\sqrt[3]{a} - 5\sqrt[3]{a^2} =$

4. Sometimes we have to simplify the radicals before we see that they are like radicals:

$$\begin{aligned} \sqrt{8y} - \sqrt{18y} &= \sqrt{\mathbf{4} \cdot 2y} - \sqrt{\mathbf{9} \cdot 2y} && \text{Simplify each radical.} \\ &= \sqrt{\mathbf{4}}\sqrt{2y} - \sqrt{\mathbf{9}}\sqrt{2y} \\ &= \mathbf{2}\sqrt{2y} - \mathbf{3}\sqrt{2y} = -\sqrt{2y} && \text{Combine like radicals.} \end{aligned}$$

Simplify and combine like terms:

a. $2\sqrt{75} + 3\sqrt{48} =$

b. $\sqrt[3]{81} + 2\sqrt[3]{24} - 3\sqrt[3]{3} =$

Activity 3 Using the Distributive Law

1. First, consider some products that do not involve the distributive law. Simplify each product as much as possible.

a. $2\sqrt{5} =$

b. $\sqrt{2}\sqrt{5} =$

c. $\sqrt{5}\sqrt{5} =$

d. $(3\sqrt{2})(2\sqrt{5}) =$

e. $(3\sqrt{5})(2\sqrt{5}) =$

f. $(2\sqrt{15})(3\sqrt{6}) =$

2. Apply the distributive law to compute the products:

a. $5(2\sqrt{3}-\sqrt{5})$

b. $4\sqrt{2}\,(4\sqrt{3}+2\sqrt{6})$

c. $(3\sqrt{x}-\sqrt{5})\,(2\sqrt{x}+3\sqrt{5})$

Activity 4 Rationalizing the Denominator

1. In some situations, it is not convenient to leave a radical in the denominator of a fraction. Explain why each of the following pairs are equal:

a. $\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$ **b.** $\dfrac{5}{\sqrt{2}}=\dfrac{5\sqrt{2}}{2}$

(Hint: Multiply top and bottom of the first fraction by the same number to get the second fraction.) Use your calculator to verify that the fractions are equal.

2. Converting a fraction to an equivalent form with no radicals in the denominator is called **rationalizing the denominator**. (It's okay to have radicals in the numerator.) For each fraction, multiply top and bottom by a number that will rationalize the denominator.

a. $\dfrac{-\sqrt{3}}{\sqrt{7}}=$ **b.** $\dfrac{10}{\sqrt{5}}=$

3. Be careful: it is easier to simplify the denominator before you rationalize it! For example:

$$\frac{6}{\sqrt{12}}=\frac{6}{2\sqrt{3}}=\frac{3}{\sqrt{3}}$$

Now rationalize the denominator:

$$\frac{3}{\sqrt{3}}\cdot\frac{\boldsymbol{\sqrt{3}}}{\boldsymbol{\sqrt{3}}}=$$

(Or better yet, recognize that $\dfrac{3}{\sqrt{3}}=\sqrt{3}$ by the definition of square root.)

a. $\sqrt{\dfrac{27x}{20}} =$

b. $\dfrac{6\sqrt{2}}{\sqrt{3v}} =$

4. To rationalize a binomial denominator, we use the **conjugate** of the binomial. For example:

Binomial	**Conjugate**
$3+\sqrt{7}$	$3-\sqrt{7}$
$\sqrt{6}-\sqrt{3}$	$\sqrt{6}+\sqrt{3}$
$\sqrt{5}+2$	?

Try multiplying each binomial above by its conjugate and see what happens:

a. $(3+\sqrt{7})\,(3-\sqrt{7}\,) =$

b. $(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3}) =$

c. $(\sqrt{5}+2)\,($? $) =$

No more radicals, right? Also, did you notice that each calculation is of the form $(a+b)(a-b)$? So you know that the product will be a^2-b^2.

5. a. Now we'll use the conjugate to rationalize a binomial denominator. Consider the fraction $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$. We multiply top and bottom of the fraction by the conjugate of the denominator:

$$\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \cdot \frac{\boldsymbol{\sqrt{3}+\sqrt{2}}}{\boldsymbol{\sqrt{3}+\sqrt{2}}} = \frac{\sqrt{3}\,\sqrt{3}+\sqrt{3}\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$$
$$= \frac{3+\sqrt{6}}{3-2} = 3+\sqrt{6}$$

Now try some yourself:

b. $\dfrac{y}{\sqrt{5}-y}$

c. $\dfrac{\sqrt{6}-3}{2-\sqrt{6}}$

d. $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Simplifying radicals
- Adding or subtracting like radicals
- Simplifying products or quotients of radicals
- Rationalizing the denominator

Check Your Understanding

1. Explain the difference between simplifying a radical and evaluating a radical.

2. Explain how to simplify a cube root. List the cubes of the first six whole numbers.

3. Compare the rules for adding radicals and multiplying radicals. Illustrate by computing $4\sqrt[3]{2x^2}+3\sqrt[3]{2x^2}$ and $4\sqrt[3]{2x^2}\cdot 3\sqrt{2x^2}$.

4. Explain why we use a conjugate to rationalize a binomial denominator.

Lesson 28 Radical Equations

Activity 1 Radical Equations

1. a. To solve a radical equation, we: (1) isolate the radical, then (2) square both sides.

Be careful when squaring a binomial! For example,

$$(x-4)^2 = (x-4)(x-4) = x^2 - 8x + 16 \qquad \textbf{FOIL it out}$$

It is **not** true that

$$(x-4)^2 = x^2 - 4^2 = x^2 - 16 \qquad \textbf{WRONG!}$$

Square some binomials. Try to do the calculation mentally.

$(x+5)^2 =$

$(w-6)^2 =$

$(2t-3)^2 =$

$(3v-4)^2 =$

b. Follow the steps to solve the equation $2x - 3 = \sqrt{4x-3}$

Square both sides.

Get zero on one side.

Solve the quadratic equation.

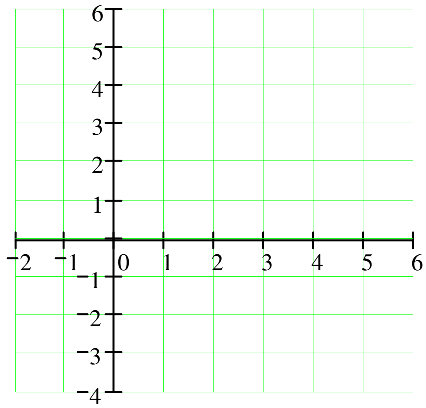
Check both solutions:

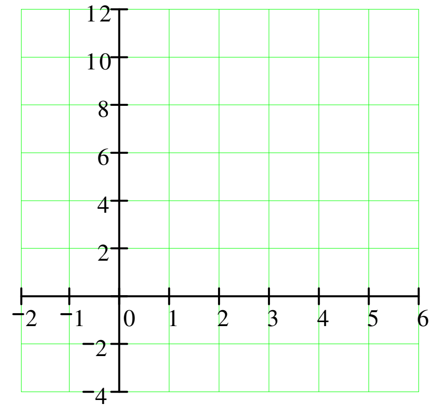
$x = 3:$

$x = 1:$

You should find that one of the "solutions" checks, and the other doesn't. Why does this happen?

c. Use a graph to solve the equation $2x - 3 = \sqrt{4x - 3}$
Graph $Y_1 = 2x - 3$ and $Y_2 = \sqrt{4x - 3}$ on the grid below left. How many solutions?





Graph $Y_1 = (2x - 3)^2$ and $Y_2 = 4x - 3$ on the grid above right. How many solutions? What happened when we squared both sides of the equation?

2. If a solution doesn't check, it is a false or **extraneous solution**. Whenever we square both sides of an equation, we must always check for extraneous solutions!
Sometimes we have to square both sides twice in order to get rid of all the radicals.

Follow the steps to solve the equation $\sqrt{y + 4} = \sqrt{y + 20} - 2$

Isolate the more complicated radical.

Square both sides.

Isolate the term with the radical.

Divide both sides by 4.

Square both sides again.

Solve for y.

Check the solution(s).

Solution:

Activity 2   Roots and Powers are Opposite Operations

1. Simplify:

a. $\sqrt{9^2} =$ $(\sqrt{9})^2 =$ $\sqrt{9}\sqrt{9} =$

b. $\sqrt{5^2} =$ $(\sqrt{5})^2 =$ $\sqrt{5}\sqrt{5} =$

c. $\sqrt[3]{8^3} =$ $(\sqrt[3]{8})^3 =$ $\sqrt[3]{8}\sqrt[3]{8}\sqrt[3]{8} =$

d. $\sqrt[3]{4^3} =$ $(\sqrt[3]{4})^3 =$ $\sqrt[3]{4}\sqrt[3]{4}\sqrt[3]{4} =$

2. Did you use your calculator? You don't need it! Explain why not.

3. Simplify. (Assume that x is a positive number.)

a. $\sqrt{x^2} =$ $(\sqrt{x})^2 =$ $\sqrt{x}\sqrt{x} =$

b. $\sqrt[3]{x^3} =$ $(\sqrt[3]{x})^3 =$ $\sqrt[3]{x}\sqrt[3]{x}\sqrt[3]{x} =$

Activity 3   Kleiber's Rule

Perhaps the single most useful piece of information a scientist can have about an animal is its metabolic rate. The resting or basal metabolic rate, BMR, is the minimum amount of energy the animal can expend in order to survive. Kleiber's rule states that the BMR for many groups of animals is given by

$$B(m) = 70m^{3/4}$$

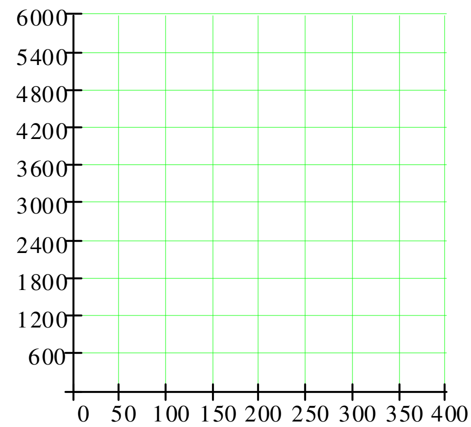
where m is the animal's mass in kilograms, and the BMR is measured in kilocalories per day.

a. Calculate the BMR for various animals whose masses are given in the table.

Animal	Bat	Squirrel	Raccoon	Lynx	Human	Moose	Rhinoceros
Weight (kg)	0.1	0.6	8	30	70	360	3500
BMR (kcal/day)							

b. Plot the data and sketch a graph of Kleiber's rule for $0 < m \leq 400$.

c. Write and solve an equation to answer the question: What is the mass of an animal whose BMR is 7500 kcal/ day?



d. A more accurate version of Kleiber's rule for mammals is

$$R(m) = 73.3\, m^{0.74}$$

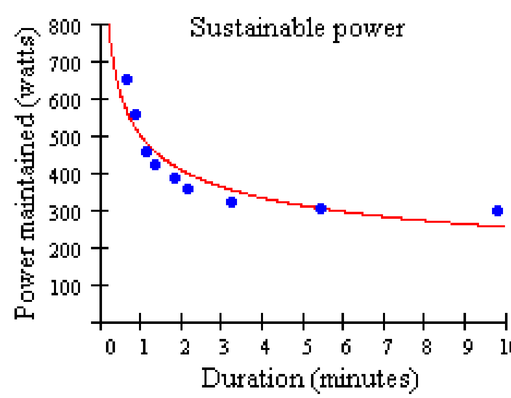
Use this formula to answer the question in part (c).

Activity 4 The Gossamer Albatross

A bicycle ergometer is used to measure the amount of power generated by a cyclist. The scatterplot shows how long an athlete was able to sustain various levels of power output. The curve is the graph of

$$y = 500x^{-0.29}$$

which approximately models the data.



a. In this graph, which variable is the input and which is the output?

b. The athlete maintained 650 watts of power for 40 seconds. What power output does the equation predict for 40 seconds?

c. The athlete maintained 300 watts of power for 10 minutes. How long does the equation predict that power output can be maintained?

d. In 1979 a remarkable pedal-powered aircraft called the Gossamer Albatross was successfully flown across the English Channel. The flight took 3 hours. According to the equation, what level of power can be maintained for 3 hours?

e. The Gossamer Albatross needed 250 watts of power to keep it airborne. For how long can 250 watts be maintained, according to the given equation?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to powers and roots:

- Solving radical equations
- Checking for extraneous solutions
- Solving formulas involving radicals
- Simplifying roots of powers

Check Your Understanding

1. What is the most important thing to remember when solving a radical equation?
2. When can extraneous solutions occur? Why?
3. Explain why it is not always true that $\sqrt{x^2} = x$.
4. Explain how to solve the equation $4x + 4 = \sqrt{3x + 4}$ by graphing.

Lesson 29 Exponential Growth and Decay

Activity 1 Population Growth

1. In a laboratory experiment, researchers establish a colony of 100 bacteria and monitor its growth. The colony triples in population every day.

a. Complete the table showing the population, $P(t)$, of bacteria t days later.

t	$P(t)$
0	100
1	
2	
3	
4	
5	

$P(0) = 100$

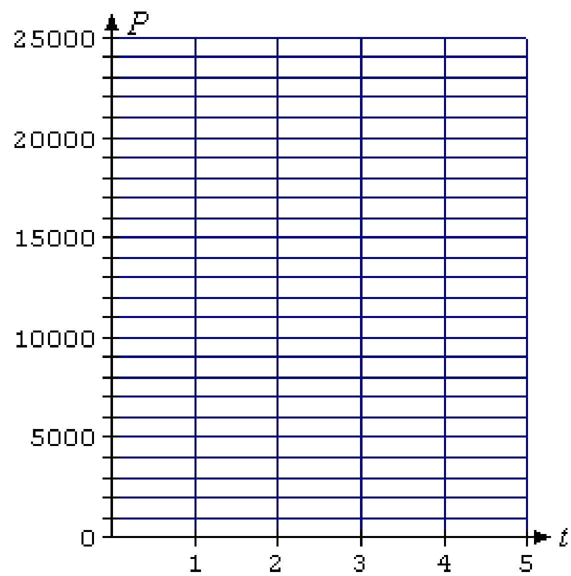
$P(1) = 100 \cdot 3 =$

$P(2) = [100 \cdot 3] \cdot 3 =$

$P(3) =$

$P(4) =$

$P(5) =$



b. Plot the data points from the table and connect them with a smooth curve.

c. Write a function that gives the population of the colony at any time, t, in days.

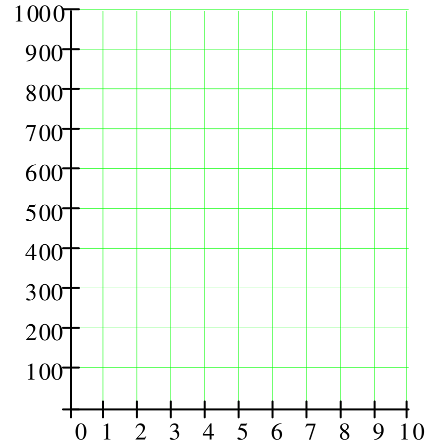
d. Graph your function from part (c) using a calculator. (Use the table to choose an appropriate window.)

e. Evaluate your function to find the number of bacteria present after 8 days.

How many bacteria are present after 36 hours?

2. A beekeeper starts a new hive with 20 insects. The population can increase in size by a factor of 1.8 every month.

a. Write a formula for the population of bees, $B(t)$, after t months.



b. How many bees will there be after 4 months?

After 6 weeks?

c. Graph your function using a calculator, then sketch the graph on the grid above.

Activity 2 Percent Increase

1. Imagine that you take a job that pays $1000 a month, and every year you'll get a 5% raise. You would like a formula that allows you to compute your salary, $S(t)$, after t years.

a. Compute your new salary after working for one year:

$$\underbrace{S(1) = 1000 + 0.05(1000)}_{\text{old salary + raise}} =$$

Compute your salary after working for two years:

$$S(2) =$$

There is a faster way to compute each year's new salary:

$$S(1) = \underbrace{1000 + 0.05(1000)}_{\text{old salary + raise}} = \underbrace{1000(1 + 0.05)}_{\textbf{factor out 1000}} = \mathbf{1000(1.05)} = 1050$$

We can compute your new salary by multiplying the old salary by 1.05. Use this method to compute your salary after working for two years:

$$S(2) = \underbrace{1050 + 0.05(1050)}_{\text{old salary + raise}} = \quad \textbf{factor out 1050}$$

b. Continue the pattern: To compute your new salary every year, multiply your old salary by 1.05. (Round to the nearest penny.)

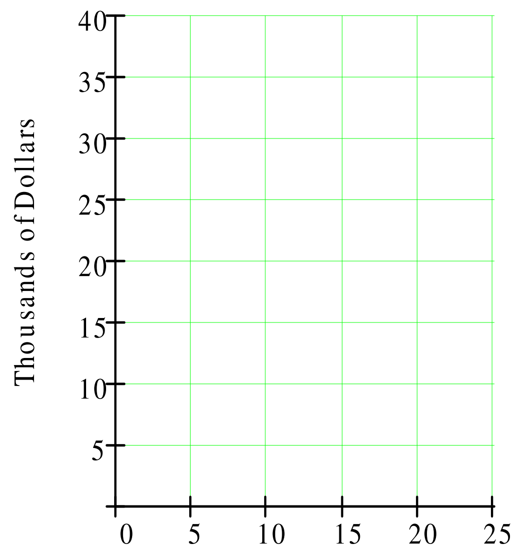
t	$S(t)$	
0	1000	
1	1050	$1000(1.05) =$
2	1102.50	$1050(1.05) = 1000(1.05)^2 =$
3		$1102.50(1.05) = 1000(1.05)^3 =$
4		

c. The table shows that $S(t)$ is an exponential function. What is its growth factor?

Write a formula for your salary $S(t)$ after t years.

2. In 1998, the average annual cost of a public college was $10,069, and costs were climbing by 6% per year.

a. Write a formula for $C(t)$, the cost of one year of college t years after 1998.



b. How much would a year of college cost in 2020?

c. Complete the table and sketch a graph of $C(t)$.

t	0	5	10	15	20	25
$C(t)$						

Activity 3 Exponential Decay

1. A small coal-mining town has been losing population since 1940, when 5000 people lived there. At each census thereafter (taken at 10-year intervals) the population declined to approximately 0.90 of its earlier figure.

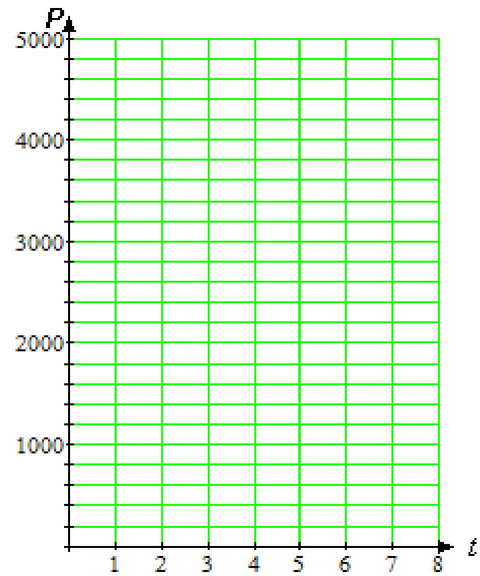
a. Complete the table showing the population, $P(t)$, of the town t decades after 1940.

t	$P(t)$
0	5000
1	
2	
3	
4	
5	

$$P(0) = 5000$$
$$P(1) = 5000 \cdot 0.90 =$$
$$P(2) = [5000 \cdot 0.90] \cdot 0.90 =$$
$$P(3) =$$
$$P(4) =$$
$$P(5) =$$

b. Plot the data points and connect them with a smooth curve.

c. Write a function that gives the population of the town at any time t in decades after 1940.



d. Graph your function from part (c) using a calculator. (Use the table to choose an appropriate window.)

e. Evaluate your function to find the population of the town in 1990.

What was the population in 1995?

2. Write a growth or decay law to describe each population, and complete the table of values.

a. A herd of 1000 deer grew by 4% per year.

t	0	1	2	3	5	10
P						

b. A herd of 1000 deer grew by 40% per year.

t	0	1	2	3	5	10
P						

c. A herd of 1000 deer decreased to 40% of its previous population every year.

t	0	1	2	3	5	10
P						

d. A herd of 1000 deer decreased by 40% each year.

t	0	1	2	3	5	10
P						

Activity 4 Comparing Linear and Exponential Decrease

In 2003, the Center for Biological Diversity filed a lawsuit against the federal government for failing to protect Alaskan sea otters. The population of sea otters, which numbered between 150,000 and 300,000 before hunting began in 1741, declined from about 20,000 in 1992 to 6000 in 2000. (Source: Center for Biological Diversity)

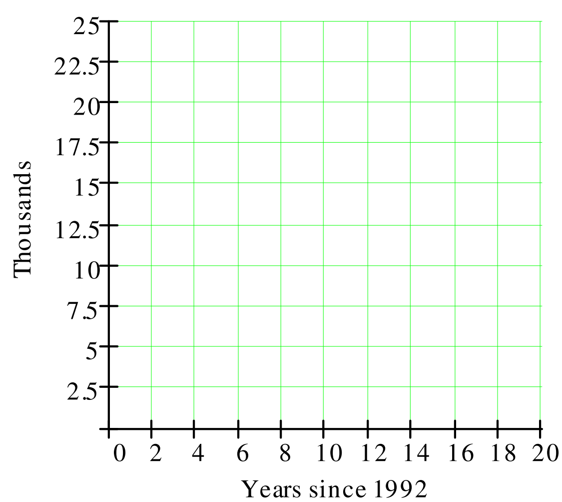
a. If the population declined **linearly** after 1992, what was its annual rate of decrease?

b. Write a formula for $L(t)$, the number of otters t years after 1992 under linear decline.

c. If the population declined **exponentially** after 1992, what was its annual decay factor? What was its annual percent decrease?

d. Write a formula for $E(t)$, the number of otters t years after 1992 under exponential decline.

e. Predict the number of sea otters in 2010 under each assumption, linear or exponential decline.



f. Graph both functions, $L(t)$ and $E(t)$, on the same grid.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to exponential growth and decay:

- Calculate percent increase or decrease
- Write a formula for exponential growth or decay
- Evaluate an exponential growth or decay function
- Find the growth factor or initial value
- Solve for percent increase or decrease
- Compare linear and exponential growth

Check Your Understanding

1. In Activity 1, how can you tell from the table of values that the bacteria population is increasing exponentially?

2. In Activity 2, what is the annual growth factor in the cost of a year of college?

3. In Activity 3, what was the decay factor for the town's population each decade? What was the percent decrease in population?

4. If the population of the town in Activity 3 decreased by 500 in the first decade, did it also decrease by 500 in the second decade? Why or why not?

Lesson 30 Exponential Functions

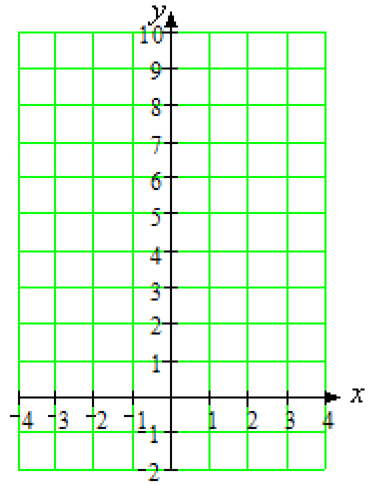
Activity 1 A New Kind of Function

1. The functions that describe exponential growth and decay are called exponential functions:

$$\boldsymbol{f(x) = a \cdot b^x, \quad b > 0,\ b \neq 1}$$

a. Make a table of values and graph $f(x) = \frac{1}{2}(2^x)$.

x	-3	-2	-1	0	1	2	3
$f(x)$							



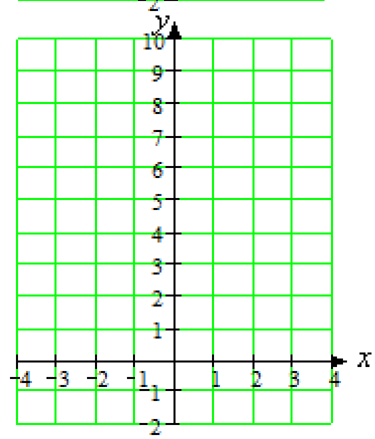
What is the y-intercept of the graph?

What is the x-intercept?

Give the equations of any asymptotes.

b. Make a table of values and graph $f(x) = 4\left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
$f(x)$							



What is the y-intercept of the graph?

What is the x-intercept?

Give the equations of any asymptotes.

2. a. What does the value of a tell you about the graph?

What does the value of b tell you about the graph?

b. Find the y-intercept of each exponential function and decide whether the graph is increasing or decreasing.

(1) $M(x) = 1.5(0.05)^x$

(2) $N(x) = 0.05(1.05)^x$

(3) $P(x) = \left(\dfrac{5}{8}\right)^x$

(4) $Q(x) = \left(\dfrac{4}{3}\right)^x$

c. Decide whether each function is an exponential function, a power function, or neither.

(1) $R(w) = 5(5)^{w-1}$ (2) $Q(w) = 2^w - w^2$

(3) $M(z) = 0.2z^{1.3}$ (4) $N(z) = z^{-3}$

Activity 2 Solving Exponential Equations

1. a. What is an exponential equation?

b. One way to solve exponential equations is to write both sides of the equation as powers with the same ______________. If two equivalent powers have the same base, then their _________________ must be equal also.

c. Solve the equation $3^{x-1} = 27^{1/2}$ by writing each side as a power of base 3.

Write 27 as a power of base 3: $3^{x-1} =$

Use the laws of exponents to simplify the right side: $3^{x-1} =$

Equate the exponents:

Solve for x:

d. Follow the steps to solve: $2^{3x-1} = \dfrac{\sqrt{2}}{16}$

Write $\sqrt{2}$ and 16 as powers of base 2: $2^{3x-1} =$

Use the laws of exponents to simplify the right side: $2^{3x-1} =$

Equate the exponents:

Solve for x:

2. An outbreak of ungulate fever can sweep through the livestock in a region so that the number of animals affected triples every 4 days.

a. A rancher discovers four cases of ungulate fever among his herd. Write a function for the number of sick cows $P(t)$ on day t.

b. Use your calculator to graph the function on the interval $0 \le t \le 20$. Sketch your graph in the space at right, and label the axes.

c. If the rancher does not act quickly, how long will it be until 324 cows are sick? Write and solve an equation to find your answer, and label the appropriate point on your graph.

Activity 3 Using Graphs and Tables

1. a. Graph $y = 2^{x+3}$ in the window:

Xmin $= -4.7$ Xmax $= 4.7$
Ymin $= 0$ Ymax $= 10$

Sketch the graph in the space at right:

b. Use your graph to solve $2^{x+3} = 5$
Round to the nearest hundredth.

2. The graph of $g(x) = P_0 b^x$ is shown in the figure.

a. Read the value of P_0 from the graph.

b. Make a short table of values for the function by reading values from the graph.

x	0	1	2
$g(x)$			

c. Does your table confirm that the function is exponential? How do you know?

d. Use your table to calculate the growth factor, b.

e. Using your answers to parts (a) and (d), write a formula for $g(x)$.

3. Which of the following tables could describe exponential functions? Explain why or why not. If the function is exponential, find its growth or decay factor.

a.

x	y
0	3
1	6
2	12
3	24
4	48

b.

t	P
0	6
1	7
2	10
3	15
4	22

c.

x	N
0	2
1	6
2	34
3	110
4	258

d.

p	R
0	405
1	135
2	45
3	15
4	5

Wrap-Up

In this Lesson, we worked on the following skills and goals related to exponential functions:

- Graph exponential functions
- Distinguish exponential functions from power functions and other functions
- Solve exponential equations with the same base
- Solve exponential equations graphically

Check Your Understanding

1. Use the order of operations to explain why $F(x) = 2(3)^x$ and $G(x) = 6^x$ are different. Confirm by evaluating $F(4)$ and $G(4)$.
2. How is the graph of $f(x) = \frac{1}{2}(2^x)$ different from the graph of $g(x) = 2^x$?
3. What is the difference between the formula for a power function and the formula for an exponential function?
4. How can you recognize an exponential function from a table of values?

Lesson 31 Logarithms

Activity 1 Solving Equations

a. Compare the two equations below. The first is a quadratic equation, and the second is an exponential equation. You can probably solve both of these equations in your head.

(1) $x^2 = 25$ (2) $2^x = 8$

Solutions:

Let's analyze how we get the solutions. In the first equation, x is squared, so we perform the opposite operation to solve, or take the square root of both sides:

$$\sqrt{x^2} = \pm\sqrt{25}$$
$$x = \pm 5$$

In the second equation, x is an exponent. We ask ourselves "What power do we need on base 2 to get 8?" Finding a power is called **taking a logarithm**.

$$\log_2(2^x) = \log_2(8)$$
$$x = 3$$

b. Now compare the two equations below. One is cubic and one is exponential:

(1) $x^3 = 17$ (2) $10^x = 135$

These are not so easy to solve in your head. But we use the same techniques as before:

$$\sqrt[3]{x^3} = \sqrt[3]{17}$$

$$\log_{10}(10^x) = \log_{10}(135)$$

But this time, we use a calculator to evaluate the answers. Locate the **log** button on your calculator, just to the left of 7.

$x =$

$x =$

The solution of the first equation answers the question "What number must I cube to get 17?" The solution of the second equation answers the question "To what power must I raise 10 to get 135?"

Activity 2 Definition of Logarithm

1. We use logarithms to solve exponential equations.

$\log_b x$	means	**"to what power must I raise b to get x?"**

Here are some examples:

a. $\log_4 64 = 3$ because $4^3 = 64$

b. $\log_2 32 =$ because

c. $\log_5 \sqrt{5} =$ because

d. $\log_{10} 0.001 =$ because

2. From the examples, we see that a logarithm is just an exponent. In fact,

$$\boxed{\mathbf{log}_b\, x = y \quad \text{if and only if} \quad b^y = x}$$

Convert each exponential equation to logarithmic form.

a. $10^x = 1000$

b. $3^x = 243$

c. $8^x = 1526$

d. $v^{5/3} = 12$

e. $10^{1.3t} = 3M_0$

f. $z^{-3t} = P$

Activity 3 Using Logarithms

a. We use **inverse operations** to solve equations. For example, cubing and taking cube roots are inverse operations. Each operation undoes the effect of the other. Start with any number, say 5, cube it, and then take the cube root. We get back to the original number:

$$5 \xrightarrow{\text{cube}} 5^3 = 125 \xrightarrow{\text{take cube root}} \sqrt[3]{125} = 5$$

We can write the two operations in one step, like this: $\sqrt[3]{5^3} = 5$. The same thing works for any number x, so $\sqrt[3]{x^3} = x$. Here are some more examples of inverse operations.

Fill in the blanks:

$(25 + 23,489) - 23,489 =$ ________

$(42 \times 13.76) \div 13.76 =$ ________

$\left(\sqrt{5936}\right)^2 =$ ________

$\dfrac{1}{1/6.97} =$ ________

b. **Logarithms and exponentials are inverse operations.** Taking a logarithm base b is the opposite of evaluating an exponential function with base b. For example, let $b = 3$, and start by start by evaluating b^x with $x = 4$. Then take the log base 3 of the result:

$$x = 4 \quad \underset{\text{exponential function}}{\rightarrow} \quad 3^4 = 81 \quad \underset{\text{logarithm}}{\rightarrow} \quad \log_3(81) = 4$$

Or: $\log_3(3^4) = 4.$ In general, we have the rule:

$$\boxed{\boldsymbol{\log_b(b^x) = x}}$$

Simplify: $\log_5 5^2 =$ ________________ $\log_6 6^8 =$ ________________

$\log_2 2^{3.56} =$ _____________ $\log_8 8^{x+4} =$ ________________

c. We use this fact to solve exponential equations.

$$\begin{aligned} 5^x &= 125 && \text{Take the log base } 5 \text{ of both sides.} \\ \log_5(5^x) &= \log_5 125 && \text{Simplify each side.} \\ x &= 3 && \text{Check the answer: } 5^{\mathbf{3}} = 125 \end{aligned}$$

Important Observation: The solution, $\log_5 125 = 3$, is the exponent in the original equation. Once again, we see that a logarithm is just an exponent; in this case it is the exponent on base 5 that will give us 125.

You can think of taking a logarithm as a command that says **find me the exponent!**

d. Logarithms base 10 are called **common logarithms**, because they are used more often in applications than logs to other bases. We often omit the base on common logs, so that

$$\log x \quad \text{means} \quad \log_{10} x$$

Evaluate the logs:

$\log 100 =$ ____________ $\log 100{,}000 =$ ________________

$\log 0.01 =$ ____________ $\log 1 =$ ________________

e. Estimate each log between two integers. Then use a calculator to evaluate the log.

$\_\_\_\_\_ < \log 1476 < \_\_\_\_\_$, $\log 1476 =$

$\_\_\_\_\_ < \log 27.9 < \_\_\_\_\_$, $\log 27.9 =$

$\_\_\_\_\_ < \log 0.694 < \_\_\_\_\_$, $\log 0.694 =$

$\_\_\_\_\_ < \log 0.0104 < \_\_\_$, $\log 0.0104 =$

Activity 4 Solving Base 10 Equations

1. We can now solve exponential equations that have base 10.

$$\begin{aligned} 10^x &= 150 && \text{Take the log base } 10 \text{ of both sides.} \\ \log_{10}(10^x) &= \log_{10}150 && \text{Simplify each side.} \\ x &= 2.1761 && \text{Check the answer: } 10^{2.1761} = 150.003 \end{aligned}$$

Logarithms are **very sensitive to round-off error.** You should keep four places of accuracy in your answers.

2. Follow the steps to solve each equation.

a. $10^{-5x} = 76$

Take logs of both sides:

Simplify each side:

Solve for x:

b. $163 = 3(10^{0.7x}) - 49.3$

Isolate the power:

Take logs of both sides:

Simplify each side:

Solve for x:

3. The atmospheric pressure decreases with altitude above the surface of the earth, according to the formula $P(h) = 30(10)^{-0.09h}$, where h is altitude in miles and P is atmospheric pressure in inches of mercury.

a. Graph this function in the window

Xmin $= 0$ Xmax $= 9.4$
Ymin $= 0$ Ymax $= 30$

Use algebra to answer the questions, then verify on your graph:

b. The elevation of Mount McKinley, the highest mountain in the United Sates, is 20,320 feet. What is the atmospheric pressure at the top? (Hint: 1 mile $= 5280$ feet.)

c. How high above sea level is the atmospheric pressure 16.1 inches of mercury?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to logarithms:

- Compute logarithms using the definition
- Convert between exponential and logarithmic form
- Solve exponential equations with base 10
- Approximate logarithms graphically

Check Your Understanding

1. What is the only difference in the steps for solving the two equations:

$$5x^{10} - 8 = 496 \quad \text{and} \quad 5(10^x) - 8 = 496$$

2. Explain how to estimate $\log_{10} x$ between two integers.

3. What happens when you apply an operation to a number, then apply the inverse operation?

4. Explain how to use the graph of $y = 3^x$ to estimate the value of $\log_3 12$.

Lesson 32 Properties of Logarithms

Activity 1 Three Properties of Logarithms
Complete the box.

Properties of Logarithms

If $x,\ y > 0$,

1. $\log_b (xy) =$

2. $\log_b \dfrac{x}{y} =$

3. $\log_b x^m =$

a. Verify that the properties are true for these examples by calculating the logs:

Property (1):

$\log_2 32 = \log_2 (4 \cdot 8) = \log_2 4 + \log_2 8$ because $2^5 = 2^2 \cdot 2^3$

________ = ______ + ______

Property (2)

$\log_2 8 = \log_2 \dfrac{16}{2} = \log_2 16 - \log_2 2$ because $2^3 = \dfrac{2^4}{2^1}$

________ = ______ − ______

Property (3)

$\log_2 64 = \log_2 (4)^3 = 3 \log_2 4$ because $(2^2)^3 = 2^6$

________ = ______ × ______

b. Given that $\log_b 2 = 0.6931,$ $\log_b 3 = 1.0986,$ and $\log_b 5 = 1.6094,$ use the properties to simplify each expression.

(1) $\log_b 3x =$

(2) $\log_b \dfrac{5}{x} =$

(3) $\log_b 8 =$

(4) $\log_b \sqrt{5x} =$

c. **Caution:** Be careful when using the properties of logarithms! Which expressions in each group are equivalent (if any)?

(1)	$\log\left(\frac{x}{25}\right)$	$\dfrac{\log x}{\log 25}$	$\log x - \log 25$	$\log (x - 25)$
(2)	$\log (25x)$	$\log 25 + \log x$	$\log (x + 25)$	$\log x \log 25$
(3)	$\log (x^2)$	$(\log x)^2$	$2 \log x$	$\log 2x$

Activity 2 Solving exponential equations

Now we can solve exponential equations to *any* base, by using the third log property.

a. Follow the steps to solve the equation $3^x = 25$

Take the log base 10 of both sides.

Apply the third log property.

Divide both sides by $\log 3$.

Use a calculator to obtain an approximation to 4 decimal places.

b. Solve $12 \cdot 5^{1.5x} = 85$

c. Starting in 1998, the demand for electricity in Ireland grew at a rate of 5.8% per year. In 1998, 20,500 gigawatts were used.
Write a formula for electricity demand in Ireland as a function of time.

When did demand reach 30,000 gigawatts?

Activity 3 Logarithmic equations

a. What is a **log equation**?

b. Follow the steps to solve the equation $\log_2(3x-1)=5$

Convert to exponential form.

Solve the new equation.

Check the solution.

c. To solve more complicated log equations, we follow three steps:

Step 1	Use the log properties to combine any expressions containing logs into a single logarithm.
Step 2	Convert the equation to exponential form.
Step 3	Solve the new equation.

Don't forget to check for extraneous solutions!

Solve: $\log_4(x+8)+\log_4(x+2)=2$

Step 1 Use log properties to write the left side as a single log.

Step 2 Convert to exponential form.

Step 3 Solve.

Check:

Wrap-Up

In this Lesson, we worked on the following skills and goals related to logarithms:

- Use the properties of logarithms to simplify expressions
- Solve exponential equations using base 10 logarithms
- Solve logarithmic equations
- Solve formulas involving exponential and logarithmic expressions

Check Your Understanding

1. True or False:

a. $\log(x+y)=\log x+\log y$

b. $\log(x-y)=\log x-\log y$

c. $\log(xy)=\log x\cdot\log y$

d. $\log\left(\frac{x}{y}\right)=\frac{\log x}{\log y}$

2. How can we use common logarithms to solve exponential equations with any base?

3. How do we check for extraneous solutions when solving a logarithmic equation?

4. How might you use the properties of logarithms when solving a logarithmic equation?

Lesson 33 Applications of Exponential Functions

Activity 1 Doubling-time

1. Lately the inflation rate in Freedonia has been 8% annually. ("Inflation" means that prices are rising by 8% per year.)

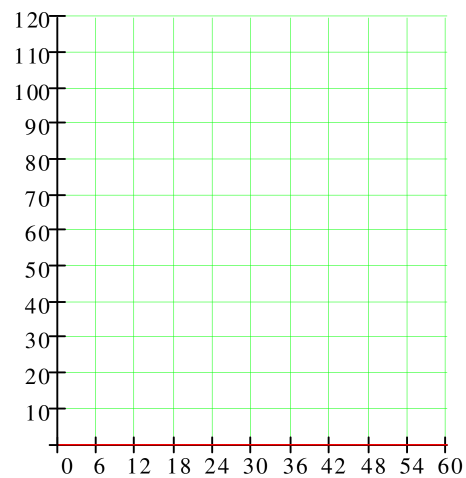
a. Write a formula for the price of a $5 item as a function of time.

b. How long will it take for the price to double from $5 to $10? (Round to the nearest year.)

c. Graph your function in the window

Xmin $= 0$, Xmax $= 94$
Ymin $= 0$, Ymax $= 120$

and sketch the curve on the grid.



d. Use the Trace to complete the table below. How long will it take the price to double from $10 to $20? From $20 to $40? (Round to the nearest year.)

t	0			
P	5	10	20	40

e. Add the horizontal lines $y = 10$, $y = 20$, and $y = 40$ to your graph. Use the **intersect** feature to verify that the price doubled from $5 to $10, from $10 to $20, and from $20 to $40 in equal periods of time.

f. Use algebra to answer the question: What is the doubling time, rounded to four decimal places?

2. Compound interest: If P dollars is invested at an annual interest rate r (expressed as a decimal) compounded n times yearly, the balance of the account A after t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

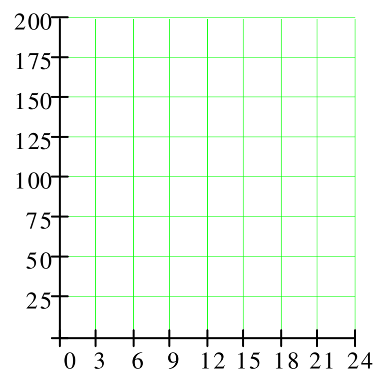
If you invest \$1000 at 10% compounded quarterly, how long will it take the balance to double?

Activity 2 Half-Life

The **half-life** of a radioactive substance is the time it takes for half of the substance to decay.

1. a. Suppose we have 200 grams of a radioactive substance whose half-life is 6 years. Complete the table and sketch a graph.

t	$N(t)$
0	
6	
12	
18	
24	



b. Write and solve an equation to find the annual decay factor for the substance in part (a). Then write its decay law.

c. What is the annual percent decay rate?

2. a. Radium-226 decays at a rate of 0.4% per year. Write a formula for $Q(t)$, the amount of radium-226 left after t years, in terms of Q_0.

b. Find the half-life, H, of radium-226. (Hint: You do not need to know the value of Q_0.)

c. Use your answer to part (a) and the formula for $Q(t)$ in terms of H to write another formula for $Q(t)$.

d. Show that your formulas in parts (a) and (c) are equivalent.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to logarithms:

- Find the doubling time or half-life of an exponential function
- Write exponential functions using doubling time and half-life

Check Your Understanding

1. Explain how to find the doubling time of an exponential function.

2. Explain how to use the doubling time to sketch the graph of an exponential function.

3. a. Write down the steps to solve the equation $1600 = 1000\left(1 + \frac{r}{4}\right)^{4t}$ for r if you know the value of t.

b. Write down the steps to solve the equation $1600 = 1000\left(1 + \frac{r}{4}\right)^{4t}$ for t if you know the value of r.

4. Delbert claims that if the half-life of an isotope is 25 years, it will all decay in 50 years. Is he correct? Why or why not?

Lesson 34 Logarithmic Scales

Activity 1 Logarithmic Scales

a. The table shows the weights of different animals. Try to plot all these values on a number line. Why is that difficult?

Animal	Shrew	Cat	Wolf	Horse	Elephant	Whale
Mass (kg)	0.004	4	80	300	5400	$70,000$

Instead, take the log of each animal's weight, then plot those values:

Animal	Shrew	Cat	Wolf	Horse	Elephant	Whale
Mass (kg)	0.004	4	80	300	5400	$70,000$
Log (Mass)						

Number line:

b. This type of graph is called a **log scale**.

Animal				
Mass (kg)	0.6	6	60	600
Log (Mass)				

What do you notice about the weights of these four animals?

What do you notice about the logs of their weights?

Multiplying by a factor of 10 on a normal scale results in __________ on a log scale.

c. **Richter scale:** On April 30, 1986, an earthquake in Mexico City measured 7.0 on the Richter scale. On September 21 a second earthquake, measuring 8.1, hit Mexico City. How many times stronger was the September quake than the one in April?

Let A_1 denote the amplitude of the April quake, with $M_1 = 7.0$
Let A_2 denote the amplitude of the September quake, with $M_2 = 8.1$

Use the formula $M = \log_{10}\left(\frac{A}{A_0}\right)$ to calculate A_1 and A_2 (in terms of A_0):

$M_1 = \log_{10}\left(\frac{A_1}{A_0}\right)$ $M_2 = \log_{10}\left(\frac{A_2}{A_0}\right)$

Finally, compute the ratio : $\frac{A_2}{A_1} =$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to logarithms:

- Plot and read values on a logarithmic scale
- Solve problems involving log scales such as pH, decibels, and the Richter scale

Check Your Understanding

1. How does a log scale differ from a linear scale?

Lesson 35 Polynomial Functions

Activity 1 Polynomial Functions

1. a. Write the definition of a **polynomial function**.

b. Give an example of a trinomial of degree four whose lead coefficient is -5.

2. Products of polynomials:

a. Multiply $(3x-2)(4x^2+x-2)$

b. Multiply $(2a^2-3a+1)(3a^2+2a-1)$

c. Find the indicated terms of the product. (Do not compute the entire product!)

$$(2-x+3x^2)(3+2x-x^2+2x^4)$$

Constant term:

Linear term:

Quadratic term:

Activity 2 Cubic Polynomials

1. Cubes of Binomials

a. Expand $(x+2)^3$ by multiplying out $(x+2)(x+2)(x+2)$

b. Complete the formulas for **cubes of binomials**:

Cubes of Binomials
$(a+b)^3 =$
$(a-b)^3 =$

Use the formula to expand: $(x+2)^3 =$

c. Use the formula to expand:

$(2-5\sqrt{t})^3 =$

2. Factoring cubics
a. Multiply $(a+b)(a^2-ab+b^2)$

Multiply $(a-b)(a^2+ab+b^2)$

Hmmm....... Interesting.

b. Write the formulas for factoring the **sum or difference of two cubes**.

Factoring the Sum or Difference of Cubes
$a^3+b^3 =$
$a^3-b^3 =$

Memorize these two formulas!

c. Factor x^3-8y^3

d. Factor $125x^3y^3+27$

Activity 3 Volume of a Box

A paper company plans to make boxes without tops from sheets of cardboard 8 inches wide and 12 inches long. They will cut out four squares of side x inches from the corners of the sheet and fold up the edges.

a. Write expressions in terms of x for the length, width, and height of the resulting box.

Length:
Width:
Height:

b. Write a formula for the volume V of the box as a function of x.

c. Graph your function in the window

Xmin $= -5$ Xmax $= 10$
Ymin $= -50$ Ymax $= 100$

d. What are the largest and smallest reasonable values for x in this situation? Explain your answer.

e. Use your answer to part (d) to graph the function in a suitable window:

Xmin $=$ Xmax $=$
Ymin $=$ Ymax $=$

Sketch the graph at right, and label scales on the axes.

f. Use your graph to find the value of x that will yield a box with maximum possible volume.

Label the corresponding point on your graph.

What is the maximum possible volume?

What are the dimensions of the box with the largest possible volume?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to polynomials:

- Compute products of polynomials
- Compute cubes of binomials
- Factor the sum or difference of cubes
- Use polynomials as models

Check Your Understanding

1. In part 2(c) of Activity 1, how many terms of the product are quadratic? How many of the terms are cubic?

2. Explain the difference between the expressions $(a+b)^3$ and a^3+b^3.

3. Compare the formulas for expanding $(a-b)^2$ and $(a-b)^3$.

4. In Activity 3, why is the polynomial for the volume of the box cubic? What are the units of volume in this application?

Lesson 36 Algebraic Fractions

Activity 1 Reducing Fractions

1. Complete the box:

To Reduce an Algebraic Fraction:

1. ________________ the numerator and denominator.

2. Divide the numerator and denominator by any ________________________.

2. Reduce the following fractions:

a. Numerator and denominator are monomials, so no factoring needed.

$$\frac{-12rs^2t}{-6rst^2}$$

b. Factor numerator and denominator before canceling!

$$\frac{5y^2-20}{2y-4}$$

$$\frac{6x^2-x-1}{2x^2+9x-5}$$

c. Use the formulas for factoring sums and differences of cubes.

$$\frac{8z^3-1}{4z^2-1}$$

d. Look for negatives of binomials.

$$\frac{2a^2-ab}{b^2-2ab}$$

$$\frac{-4b^2-2b}{4b^3-b}$$

3. Which expressions are equivalent to the given fraction?

$\dfrac{3}{5}$ **a.** $\dfrac{3x}{5x}$ **b.** $\dfrac{3-x}{5-x}$ **c.** $\dfrac{30+x}{x+50}$ **d.** $\dfrac{30x}{50x}$

$\dfrac{v-1}{v+1}$ **a.** -1 **b.** $\dfrac{1-v}{1+v}$ **c.** $-\dfrac{1-v}{v+1}$ **d.** $\dfrac{v+1}{v-1}$

$\dfrac{2w-6}{w}$ **a.** -4 **b.** $\dfrac{6-2w}{-w}$ **c.** $2-\dfrac{6}{w}$ **d.** $-\dfrac{2w+6}{w}$

$\dfrac{3a+2}{3a-2}$ **a.** $\dfrac{3a-2}{3a+2}$ **b.** $\dfrac{3a-2}{2-3a}$ **c.** -1 **d.** $\dfrac{2a+3}{2a-3}$

Activity 2 Horizontal Asymptote

Envirogreen Technology, Inc. decides to produce a new type of water filter for home use. They spend \$5000 for startup costs, and each filter costs \$50 to produce.

a. To help her decide on a selling price for the filters, the marketing manager would like to know the average cost per filter if they produce x filters.

(i) Compute the total cost of producing x filters:

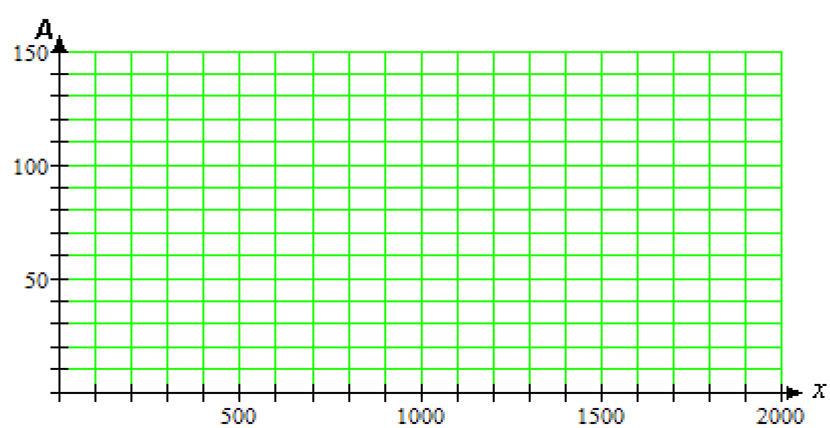
$$\text{Total Cost} = \text{Startup Cost} + \text{Cost of } x \text{ Filters}$$
$$=$$

(ii) Divide the total cost by the number of filters produced.

$$\text{Average Cost} = \frac{\text{Total Cost}}{\text{Number of Filters}} =$$

b. Sketch a graph of the average cost, A.

x	A
50	
100	
200	
400	
500	
1000	
1250	
2000	



Use the graph and the table to help you answer the questions.

c. Approximately how many filters should Envirogreen produce for the average cost to be $75?

d. What happens to the average cost per filter as x increases?

e. As x increases, the average cost appears to be approaching a limiting value. What is that value?

f. A market analysis concludes that Envirogreen can sell 1250 filters this year. How much should they charge for one filter if they would like to make a total profit of $100,000? Use the formula
Profit = Revenue − Cost.

Activity 3 Vertical Asymptote

The fastest fish in the sea is the bluefin tuna, which has been clocked at 43 miles per hour in short sprints. A school of tuna is migrating a distance of 200 miles at an average speed of 36 miles per hour in still water, but they have run into a current flowing against their direction of travel.

a. Express the tuna's travel time t as a function of the current speed, v.

b. Complete the table showing the travel time for various current speeds.

v	0	4	8	12	16	20	24	28	32	36
t										

What happens to the travel time as the current increases?

c. Use the table to choose an appropriate window and graph your function $t(v)$. Sketch the graph and label scales on the axes.

d. Use your graph to estimate the speed of the current if the tuna's travel time was 20 hours. Show your method on the graph.

e. Give the equations of any horizontal or vertical asymptotes. What does the vertical asymptote mean in the context of the problem?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to algebraic fractions:

- Reduce fractions
- Use rational functions as models
- Interpret horizontal and vertical asymptotes

Check Your Understanding

1. What is the most important thing to remember about reducing fractions?

2. Explain why $\dfrac{x-t}{t-x} = -1$.

3. In Activity 2, why does the average cost per filter never decrease below $50?

4. What would negative values of v mean in Activity 3?

Lesson 37 Operations on Algebraic Fractions

Activity 1 Multiplying and Dividing Fractions

1. Complete the box:

To Multiply Algebraic Fractions:
1. ________________ each numerator and denominator.
2. Divide out any factors that appear in ____________________.
3.

2. Practice multiplying fractions:

a. $\dfrac{3y}{4xy-6y^2}\cdot\dfrac{2x-3y}{12x}$

b. $\dfrac{9x^2-25}{2x-2}\cdot\dfrac{x^2-1}{6x-10}$

c. $\dfrac{3x^2-7x-6}{2x^2-x-1}\cdot\dfrac{2x^2-9x-5}{3x^2-13x-10}$

3. Complete the box:

To Divide Algebraic Fractions:
1. Take the _________________ of the second fraction.
2. ___________________ each numerator and denominator.
3. Divide out any factors that appear in ____________________.
4.

4. Practice dividing fractions:

a. $\dfrac{a^2+2a-15}{a^2+3a-10}\div\dfrac{a^2-9}{a^2-9a+14}$

b. $\dfrac{8x^3 - y^3}{x + y} \div \dfrac{2x - y}{x^2 - y^2}$

Activity 2 Adding and Subtracting Fractions

1. Like Fractions

a. What are **like fractions**?

b. Complete the box:

To add or subtract like fractions, we ______________________ their numerators and ________________ the denominator.

c. Practice adding like fractions:

$$\frac{2}{a-3b} - \frac{b-2}{a-3b} + \frac{b}{a-3b} =$$

2. Unlike Fractions

a. To add unlike fractions, we must first find ____________________________ .

b. Complete the box:

To Add or Subtract Unlike Fractions
1.
2.
3.
4.

c. Follow the steps to add: $\dfrac{x-2}{4x} + \dfrac{2x-3}{3x}$

Step 1 The LCD is: $3 \cdot 4 \cdot x =$ 12X

Step 2 Find the building factor for each fraction and build:

$$\frac{3}{3} \cdot \frac{x-2}{4x} + \frac{2x-3}{3x} \cdot \frac{4}{4} = \frac{\qquad}{12x} + \frac{\qquad}{12x}$$

Step 3 Combine the numerators: $\dfrac{\qquad}{12x} =$

Step 4 Can the answer be reduced?

d. If neither denominator can be factored, the LCD is just their product.

Follow the steps to subtract: $\dfrac{2x}{3x+1} - \dfrac{x}{x-2}$

Step 1 The LCD is:

Step 2 Find the building factor for each fraction and build:

$$\frac{2x}{3x+1} - \frac{x}{x-2} = \frac{\qquad}{(3x+1)(x-2)} - \frac{\qquad}{(3x+1)(x-2)}$$

Step 3 Combine the numerators: $\dfrac{\qquad}{(3x+1)(x-2)} =$

Step 4 Can the answer be reduced?

3. Finding the LCD

a. The LCD must be divisible by each of the given ________________ .

The first step in finding the LCD is to ______________ each denominator.

b. Follow the steps to subtract: $\dfrac{2}{3y+6} - \dfrac{3}{2y+4}$

Step 1 Factor each denominator

$3y + 6 =$

$2y + 4 =$

The LCD is: 2 · 3 (Y + 2)

Step 2 Find the building factor for each fraction and build:

$$\frac{2}{2}\cdot\frac{2}{3(y+2)}-\frac{3}{2(y+2)}\cdot\frac{3}{3}=\frac{\qquad}{\text{LCD}}-\frac{\qquad}{\text{LCD}}$$

Step 3 Combine the numerators: $\frac{\qquad}{6(y+2)}$

Step 4 Can the answer be reduced?

c. Follow the steps to subtract: $\frac{x+1}{x^2+2x}-\frac{x-1}{x^2-3x}$

Step 1 Factor each denominator

$x^2+2x=$

$x^2-3x=$

The LCD is:

Step 2 Find the building factor for each fraction and build:

$$\frac{?}{?}\cdot\frac{x+1}{x(x+2)}-\frac{x-1}{x(x-3)}\cdot\frac{?}{?}=\frac{\qquad}{\text{LCD}}-\frac{\qquad}{\text{LCD}}$$

Step 3 Combine the numerators: $\frac{\qquad}{x(x+2)(x-3)}$

Step 4 Can the answer be reduced?

Activity 3 Applications

River Queen Tours offers a round-trip excursion on the Mississippi River by paddle wheel, 25 miles each way. The current in the Mississippi is 8 miles per hour.

	D	R	T
upstream			
downstream			

a. Express the time required for the downstream journey as a function of the speed of the boat, v, in still water.

b. Express the time required for the return trip upstream in terms of v.

c. By adding your answers for parts (a) and (b), write an expression for the total time for the round trip in terms of v. Then simplify the expression.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to algebraic fractions:

- Multiply and divide algebraic fractions
- Add and subtract like fractions
- Find a lowest common denominator
- Add and subtract unlike fractions

Check Your Understanding

1. If you divide 4 cakes into $\frac{1}{3}$'s, how many pieces will you have? If you divide 3 cakes into $\frac{1}{5}$'s, how many pieces will you have? How are these calculations related to the rule for dividing fractions?

2. How would you simplify $\left(\dfrac{x-3}{3x}\right)^{-2}$?

3. Compute $\dfrac{2a}{a-2b}-\dfrac{4b}{a-2b}$ and $\dfrac{2a}{a-2b}\div\dfrac{4b}{a-2b}$. Compare the methods for each computation.

4. Why do we build fractions when adding or subtracting them?

Lesson 38 More Operations on Fractions

Activity 1 Complex Fractions

a. What is a complex fraction?

b. Complete the box:

To Simplify a Complex Fraction:
1.
2.
3.

c. Follow the steps to simplify $\dfrac{\frac{2}{y}+\frac{1}{2y}}{y+\frac{y}{2}}$

Step 1 Find the LCD for $\frac{2}{y}$, $\frac{1}{2y}$, $\frac{y}{1}$, and $\frac{y}{2}$: LCD =

Step 2 Multiply each term of the top and each term of the bottom by the LCD:

$$\frac{\frac{2y}{1}\cdot\frac{2}{y}+\frac{1}{2y}\cdot\frac{2y}{1}}{\frac{2y}{1}\cdot y+\frac{y}{2}\cdot\frac{2y}{1}}=$$

Step 3 Reduce:

d. Sometimes complex fractions are written with **negative exponents**. For example,

$$\frac{x^{-1}-y}{x^{-1}}=\frac{\frac{1}{x}-y}{\frac{1}{x}}=$$

Simplify this complex fraction.

Activity 2 Application

1. The owner of a print shop volunteers to produce flyers for his candidate's campaign. His large printing press can complete the job in 4 hours, and the smaller model can finish the flyers in 6 hours. How long will it take to print the flyers if he runs both presses simultaneously?

a. Write fractions of the job can each press can complete in 1 hour.

Large press: Small press:

b. What fraction of the job can be completed in 1 hour with both presses running simultaneously?

c. How long will it take to complete the job with both presses running?

2. Now we'll try the same problem with variables. Suppose that the large press can complete a job in t_1 hours and the smaller press takes t_2 hours. (Use your answers to part (1) as a guide.)

a. Write expressions for the fraction of a job that each press can complete in 1 hour.

Large press: Small press:

b. Write an expression for the fraction of a job that can be completed in 1 hour with both presses running simultaneously.

c. Write a complex fraction for the time needed to complete the job with both presses running.

d. Write your answer to part (c) as a simple fraction.

e. Use your formula to verify your answer to part (1).

Activity 3 Polynomial Division

If an algebraic fraction cannot be reduced, we can simplify it by dividing the denominator into the numerator.

1. Monomial divisor. Divide the monomial into each term of the numerator.

$$\frac{2m^6 - 15m^4 + 7}{-5m^3}$$

2. Binomial divisor. Follow the procedure for long division. First, prepare the fraction:

Write the fraction $\dfrac{\text{numerator}}{\text{denominator}}$ this way: $\text{denominator}\,\overline{)\,\text{numerator}}$

So the *denominator* becomes the *divisor*. Write both polynomials in descending powers of the variable, and insert terms with 0 coefficient if any powers are missing.

Steps for polynomial division:
1. Divide the first term of the numerator by the first term of the denominator. Write the answer above the quotient bar.
2. Multiply the quotient by the divisor. Write the answer below the numerator.
3. Subtract Step 2 from the numerator.
4. Repeat Step 1, using the answer to Step 3 in place of the numerator.

Practice division by a binomial:

a. $\dfrac{2x^3 - 3x^2 - 2x + 4}{x + 1}$

b. $\dfrac{7 - 3t^3 - 23t^2 + 10t^4}{2t + 3}$

Wrap-Up

In this Lesson, we worked on the following skills and goals related to algebraic fractions:

- Simplify complex fractions
- Simplify expressions involving negative exponents
- Use polynomial division to simplify fractions

Check Your Understanding

1. In Activity 1, how do we use an LCD to simplify a complex fraction?
2. Which of the five laws of exponents apply to sums and differences?
3. In parts 1 and 2 of Activity 2, how did we obtain the answer to (c) from the answer to (b)?
4. In part 2(b) of Activity 3, how did you rewrite the numerator before starting to divide?

Lesson 39 Equations with Algebraic Fractions

Activity 1 Proportions

a. What is a **proportion ?**

To clear the fractions from a proportion, we **multiply each side by both denominators**.

b. Clear the denominators from the proportion: $\dfrac{-3}{4} = \dfrac{y-7}{y+14}$

$$\_\_\_\_\_\_\_\_\_\_ \cdot \frac{-3}{4} = \frac{y-7}{y+14} \cdot \_\_\_\_\_\_\_\_\_\_$$

What is the name of the short-cut for clearing the denominators? ________________________   This short-cut works **only** for solving proportions -- **do not try to use it on any other equations, or any other fraction problems!**

c. Solve $\dfrac{30}{r} = \dfrac{20}{r-10}$

d. The Wildlife Commission tags 30 Canada Geese at one of its migratory feeding grounds. When the geese return, the commission captures 45 geese, of which four are tagged. What is the commission's estimate of the number of geese that use the feeding ground?

Step 1 Choose a variable: what is the unknown quantity?

Step 2 Set up a proportion: write two ratios, one for the entire population of geese, and one for the captured geese.

	all geese		captured geese
$\dfrac{\text{tagged geese}}{\text{total geese}}$:		$=$	

Step 3 Solve the proportion.

Activity 2 Graphical Solution

A florist finds that she will sell $\dfrac{300}{x}$ dozen roses per week if she charges x dollars for a dozen. Her suppliers will sell her $5x - 55$ dozen roses if she sells them at x dollars per dozen.

a. Graph the **demand function**, $D(x) = \dfrac{300}{x}$,
and the **supply function**, $S(x) = 5x - 55$,
in the window

Xmin $= 0$ Xmax $= 30$
Ymin $= 0$ Ymax $= 40$

Sketch your graph at right, labeling the axes.
At what **equilibrium price,** x, will the florist sell all the roses she purchases?

b. Write and solve an equation to verify the equilibrium price.

Activity 3 Equations with Fractions

1. a. If an equation contains more than one fraction, we can clear all the denominators at once by multiplying both sides by the ____________ of the fractions.

b. Follow the steps to solve $\dfrac{2}{x+1} + \dfrac{1}{3x+3} = \dfrac{1}{6}$

Step 1 Find the LCD. First factor each denominator:

$x + 1 =$

$3x + 3 =$

$6 =$

LCD $=$

Step 2 Multiply **each** term of the equation by the LCD. If you write each denominator in its factored form, it will be easier to simplify:

$$\frac{6(x+1)}{1} \cdot \frac{2}{x+1} + \frac{1}{3(x+1)} \cdot \frac{6(x+1)}{1} = \frac{1}{6} \cdot \frac{6(x+1)}{1}$$

Step 3 Simplify by reducing each term:

Step 4 Solve:

2. Extraneous Solutions

a. An apparent solution is **extraneous** if it causes one or more of the denominators in the equation to be ________________.

b. Consider the equation $\dfrac{1}{x-1}+\dfrac{2}{x+1}=\dfrac{x-3}{x^2-1}$

Which values of x cause one or more of the denominators to be zero?

$x =$

c. Follow the steps to solve $\dfrac{1}{x-1}+\dfrac{2}{x+1}=\dfrac{x-3}{x^2-1}$

Step 1 Find the LCD. First factor each denominator:

$x-1=$

$x+1=$

$x^2-1=$

LCD $=$

Step 2 Multiply **each** term of the equation by the LCD. If you write each denominator in its factored form, it will be easier to simplify:

$$\frac{\qquad}{\mathbf{1}}\cdot\frac{1}{x-1}+\frac{2}{x+1}\cdot\frac{\qquad}{\mathbf{1}}=\frac{x-3}{(x+1)(x-1)}\cdot\frac{\qquad}{\mathbf{1}}$$

Step 3 Simplify by reducing each term:

Step 4 Solve:

Step 5 Check: Are any of the solutions extraneous?

3. Solving Formulas

Follow the steps to solve $\frac{1}{R} = \frac{1}{A} + \frac{1}{B}$ for B.

Step 1 Find the LCD of all the fractions in the equation. LCD $=$

Step 2 Multiply **each** term by the LCD:

$$\frac{}{1} \cdot \frac{1}{R} = \frac{1}{A} \cdot \frac{}{1} + \frac{1}{B} \cdot \frac{}{1}$$

Step 3 Simplify.

Step 4 Get all terms with B on one side.

Step 5 Now here is the new part: if you have two terms containing the variable (B in this case), you should **factor out the variable:**

Step 6 Finally, solve for B:

Wrap-Up

In this Lesson, we worked on the following skills and goals related to algebraic fractions:

- Solve equations involving algebraic fractions
- Check for extraneous solutions
- Solve formulas involving algebraic fractions

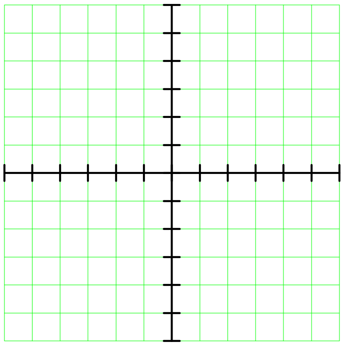
Check Your Understanding

1. What is the first step in solving an equation that includes algebraic fractions?
2. If the equation also contains terms without fractions, should you multiply those terms by the LCD?
3. What are extraneous solutions, and when might they arise?
4. If you are solving a formula and two or more terms contain the variable you are solving for, what should you do?

Lesson 40 Parallel and Perpendicular Lines

Activity 1 Horizontal and Vertical Lines

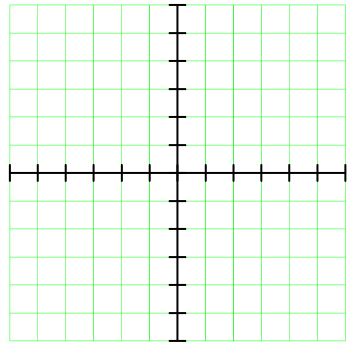
a. Graph $3y = 15$.
Compute its slope.



$m =$

Illustrate Δx and Δy on your graph.

b. Graph $2x = -4$.
Compute its slope.



$m =$

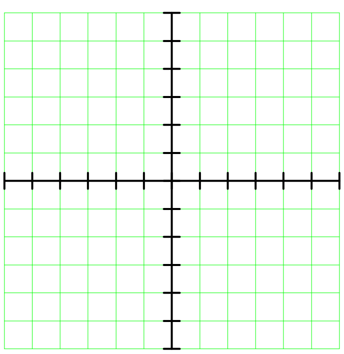
Illustrate Δx and Δy on your graph.

The slope of a horizontal line is ________________________ .

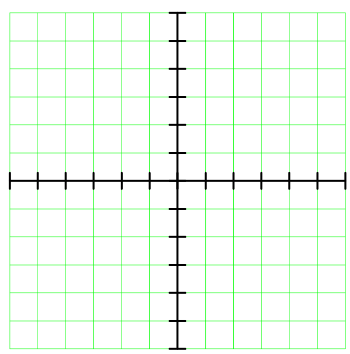
The slope of a vertical line is __________________________ .

c. Make a sketch of each line, then find its equation.

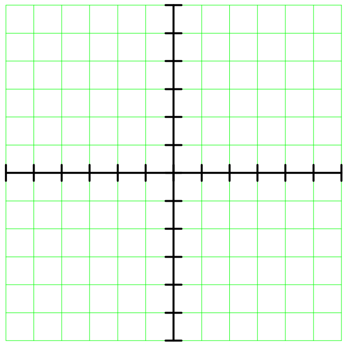
(1) A horizontal line through $(2, -4)$



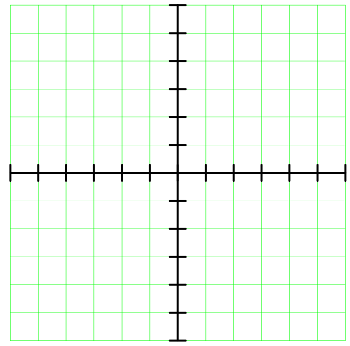
(2) The y-axis



(3) Parallel to the y-axis and including $(-1,\ -2)$



(4) Perpendicular to $x = 3$, intersecting it at $(3,\ 5)$



Activity 2 Parallel and Perpendicular Lines

1. The slopes of several lines are given below. Which of the lines are parallel to the graph of $y = 2.5x - 3$, and which are perpendicular to it? Which are neither? Support each answer with a calculation.

a. $m = \dfrac{2}{5}$

b. $m = \dfrac{25}{10}$

c. $m = \dfrac{-8}{20}$

d. $m = \dfrac{-45}{18}$

e. $m = \dfrac{40}{16}$

f. $m = 25$

g. $m = \dfrac{-1}{25}$

h. $m = \dfrac{-5}{10}$

2. In each part, determine whether the two lines are parallel, perpendicular, or neither.

a. $2x - 7y = 14$; $7x - 2y = 14$

b. $x + y = 6$; $x - y = 6$

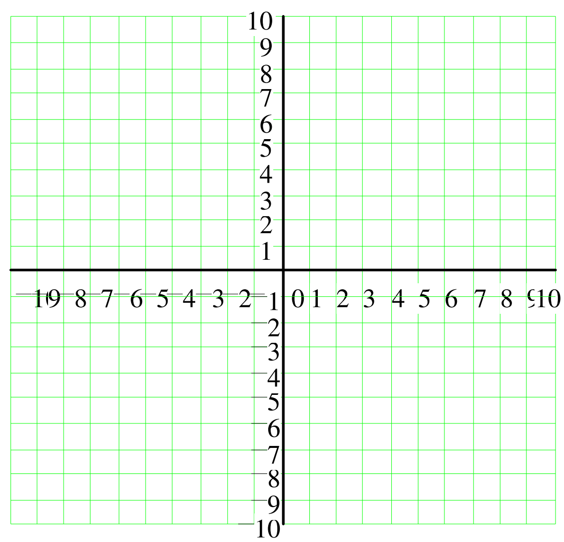
c. $x = -3$; $3y = 5$

d. $2y = 5$; $5y = -2$

Activity 3 Applications

1. a. Sketch the triangle with vertices

$$A(-6, -3),\ B(-6, 3),\ \text{and}\ C(4, 5)$$

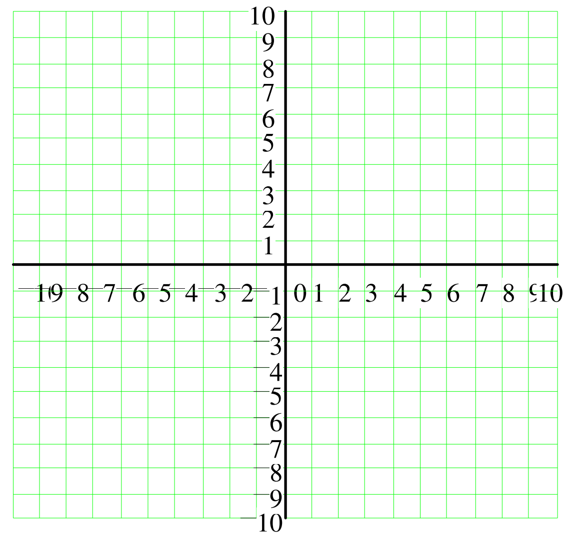


b. Find the slope of the side $\overline{AC}$.

c. Sketch the altitude from point B to side $\overline{AC}$. Find the slope of the altitude.

d. Find an equation for the line that includes the altitude from point B to side $\overline{AC}$. (Hint: You know a point (x, y) that lies on the line.)

2. **a.** Write the equation $2y - 3x = 5$ in slope-intercept form, and graph the equation.

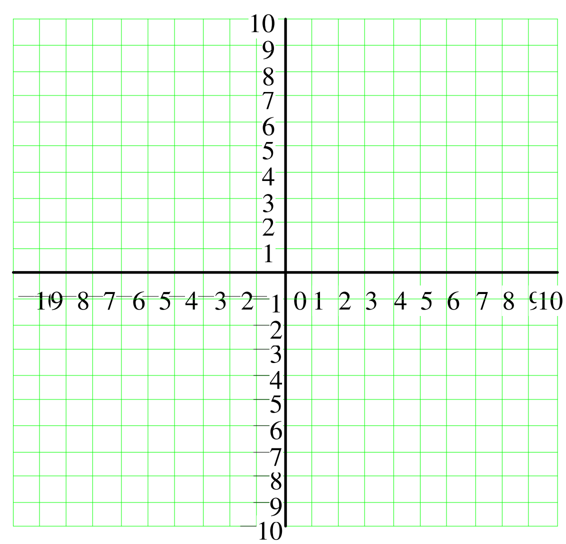


b. What is the slope of any line that is parallel to $2y - 3x = 5$?

c. On your graph for part (a), sketch by hand a line that is parallel to $2y - 3x = 5$ and passes through the point $(-3, 2)$.

d. Use the point-slope formula to write an equation for the line that is parallel to the graph of $2y - 3x = 5$ and passes through the point $(-3, 2)$.

3. **a.** Write the equation $x - 2y = 5$ in slope-intercept form, and graph the equation.



b. What is the slope of any line that is perpendicular to $x - 2y = 5$?

c. On your graph for part (a), sketch a line that is perpendicular to $x - 2y = 5$ and passes through the point $(4, -3)$.

d. Use the point-slope formula to write an equation for the line that is perpendicular to the graph of $x - 2y = 5$ and passes through the point $(4, -3)$.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to equations and graphs:

- Find equations for horizontal and vertical lines
- Decide whether two lines are parallel, perpendicular, or neither
- Find equations for parallel or perpendicular lines

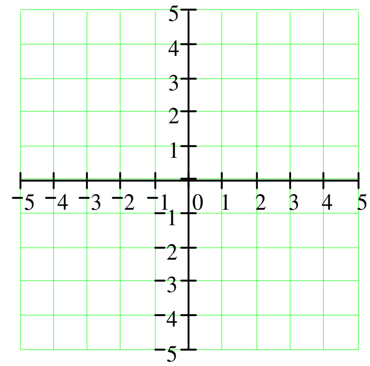
Check Your Understanding

1. How can you calculate the reciprocal of a decimal number?
2. Compare the graphs of the lines $y = \frac{2}{5}$ and $y = \frac{2}{5}x.$
3. What two things do you need to know about a line in order to write its equation?
4. If you know the slope of the base of a triangle, how do you find the slope of its altitude?

Lesson 41 Conic Sections: Ellipses

Activity 1 Central Ellipses

a. Consider the equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$



Complete the table of values and plot the points.

$\boldsymbol{x}$	0		±1	
$\boldsymbol{y}$		0		±1

Draw a smooth curve through the points to make an **ellipse**.

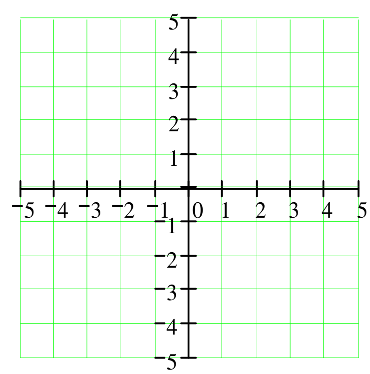
b.

The **standard form** for the equation of an ellipse centered at the origin is

Sketch a **central ellipse** at right. Label the **major axis, minor axis, vertices,** and **covertices**.

c. Follow the steps to graph the ellipse $3x^2 + 4y^2 = 36$

Step 1 Divide both sides of the equation by 36 and simplify to get the standard form.



Step 2 Identify the values of a and b. (Use your calculator to approximate if necessary.)

$a =$
$b =$

Step 3 Plot the vertices and covertices. Sketch the ellipse through those points.

Activity 2 Translated Ellipses

a.

The **standard form** for an ellipse centered at (h, k) is

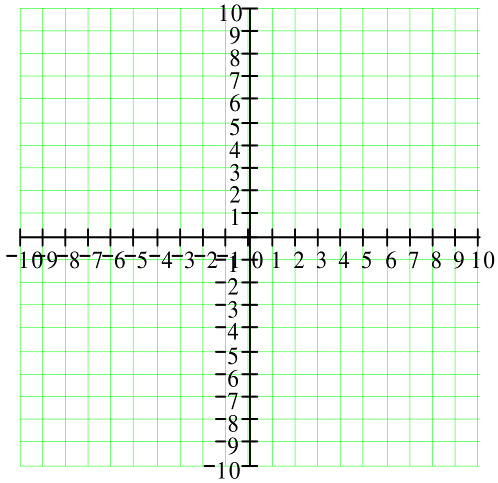
b. Follow the steps to graph $\dfrac{(x-2)^2}{4} + \dfrac{(y-5)^2}{25} = 1$

Step 1 Identify the center and the values of a and b.

$(h,\ k) =$

$a =$

$b =$

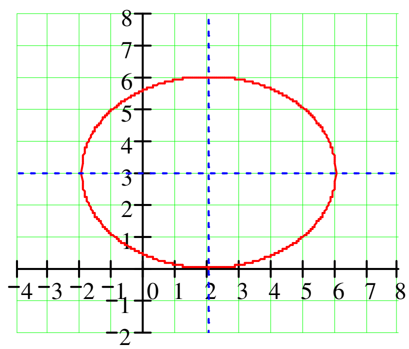


Step 2 Plot the center. Use the values of a and b to locate the vertices and covertices.

Step 3 Sketch an ellipse through the vertices and covertices.

Activity 3 Writing the Equation for an Ellipse

a. To write the equation of an ellipse, we need to know its center (h, k) and the values of a and b.



The center of the ellipse is

$(h, k) =$

The length of the horizontal axis is

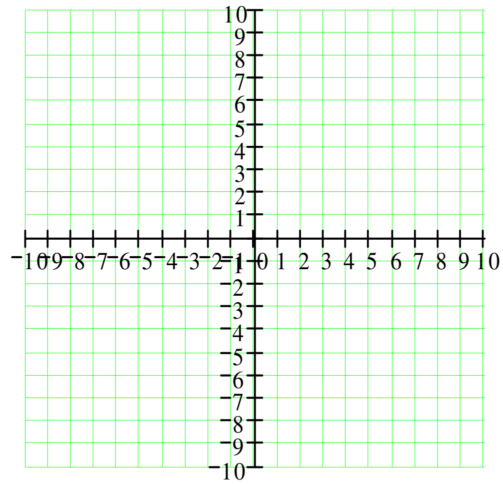
$2a =$ __________, so $a =$ ______________

The length of the vertical axis is

$2b =$ __________, so $b =$ ______________

The equation of the ellipse is

b. Follow the steps to find the equation of the ellipse with vertices at $(-3,-5)$ and $(9,-5)$, covertices at $(3,0)$ and $(3,-10)$.



Step 1 Plot the given points and sketch the ellipse.

Step 2 Locate the center of the ellipse.

$(h,k) =$

Step 3 Find the values of a and b.

$a =$ $b =$

Step 4 Write the equation of the ellipse:

Activity 4 The General Quadratic Equation

a. The **general quadratic equation** in two variables is

$$\boldsymbol{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$$

We will only consider quadratic equations in which $B = 0$. The graphs of these equations are **conic sections:** Name the four conic sections:

(1) (2)

(3) (4)

If $A = C$, the graph is ____________________. If A and C have the same sign (either both positive or both negative), the graph is ________________________.

b. We convert an equation in general form to standard form by completing the square. Follow the steps to write the equation below in standard form:

$$6x^2 + 5y^2 - 12x + 20y - 4 = 0$$

Step 1 Collect the x-terms and y-terms together: $= 4$

Step 2 Factor out the coefficients of x^2 and y^2. $= 4$

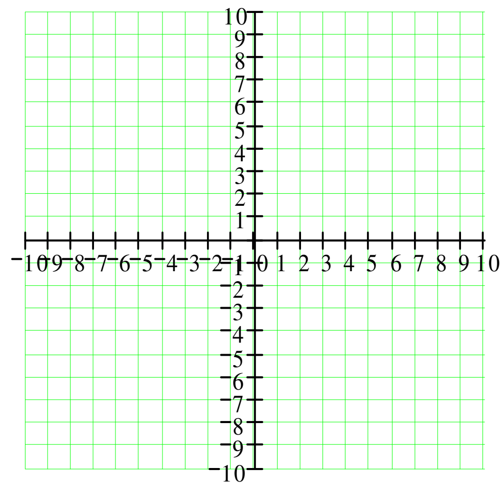
Step 3 Complete the square in x and in y.

***** Be careful! ***** Notice that you have really added $6(1) = 6$ to the x-terms and $5(4) = 20$ to the y-terms on the left side of the equation, so you must add $6 + 20$ to the right side of the equation as well.

Step 4 Simplify each side.

Step 5 Divide both sides by the constant, and simplify.

Graph the ellipse on the grid.



Wrap-Up

In this Lesson, we worked on the following skills and goals related to conic sections:

- Write the equation for an ellipse in standard form
- Graph an ellipse
- Find the equation for an ellipse

Check Your Understanding

1. In Activity 1, how did you use a and b to graph an ellipse?
2. What does it mean for an ellipse to be translated?
3. In Activity 3, how did you find a and b?
4. What was the Caution in Activity 4?

Lesson 42 Conic Sections: Hyperbolas

Activity 1 Central Hyperbolas

a. Write the two standard equations for **central hyperbolas** in the box.

Central Hyperbolas

1. (opens left and right)

2. (opens up and down)

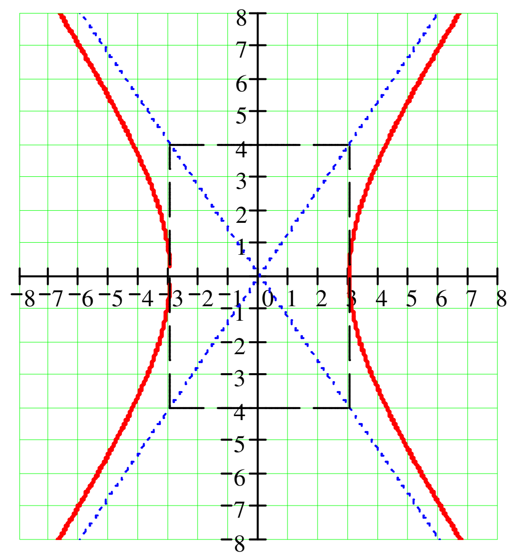
The segment joining the two vertices is called the ______________________.

The perpendicular segment is called the ______________________________.

b. How to graph a hyperbola: Let's consider an example, $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

Complete the table.

x	y
0	
	0



(1) Plot the vertices.

(2) Plot the two points $(0, b)$ and $(0, -b)$. These points are not on the hyperbola -- we will use them, along with the vertices, to sketch the **central rectangle**.

(3) Sketch the **asymptotes** through the diagonals of the central rectangle.

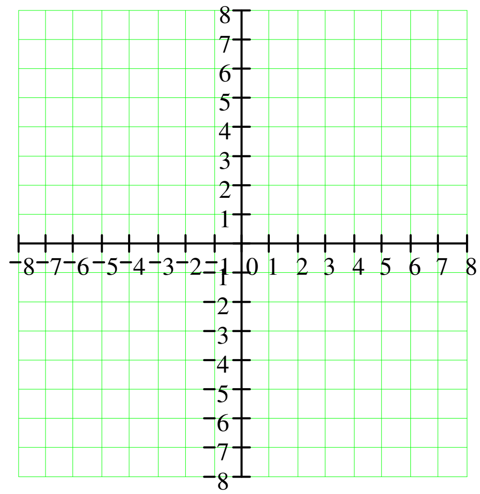
(4) Finally, sketch the two **branches** of the hyperbola through the vertices and approaching the asymptotes. (Neither the central rectangle nor the asymptotes are part of the graph; they are just tools.)

c. Follow the steps to graph $-4x^2 + 25y^2 = 100$

Step 1 Write the equation in standard form: divide both sides by 100.

Step 2 Identify the values of a and b.

$a =$ $b =$



Step 3 Plot the vertices $(0, b)$ and $(0, -b)$, and the two points $(a, 0)$ and $(-a, 0)$.

Step 4 Sketch the central rectangle.

Step 5 Sketch the asymptotes.

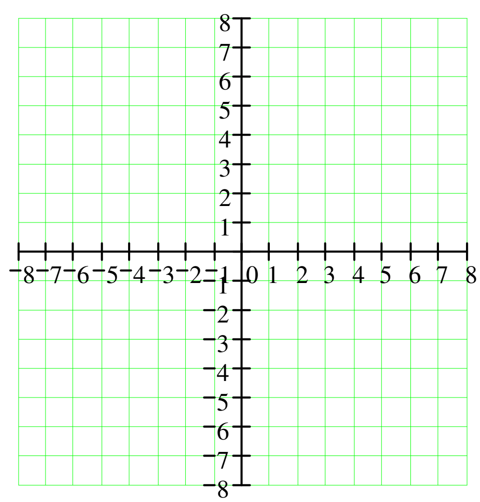
Step 6 Sketch the hyperbola.

Activity 2 Translated Hyperbolas

a. Complete the box:

The **standard forms** for a hyperbolas centered at (h, k) are
1.
2.

b. Follow the steps to graph $\dfrac{(x-3)^2}{4} - \dfrac{(y+1)^2}{9} = 1$



Step 1 Identify the values of the center $(h,\, k)$, and a and b.

$(h, k) =$

$a =$ $b =$

Step 2 Plot the vertices and the ends of the conjugate axis.

Step 3 Sketch the central rectangle.

Step 4 Sketch the asymptotes.

Step 5 Sketch the hyperbola.

c. Find the equations of the asymptotes. The asymptotes pass through the center of the hyperbola, so

$$(x_1,\, y_1) = (h, k) =$$

Look at the central rectangle to see that the slopes of the asymptotes are

$$m = \frac{\Delta y}{\Delta x} = \frac{\pm b}{a} =$$

Substitute these values into the **point-slope form** and find the equations of the asymptotes:

d. On the same grid above, sketch the graph of $\dfrac{(y+1)^2}{9} - \dfrac{(x-3)^2}{4} = 1$

How does the graph differ from the graph in part (b)?

Finally, on the same grid sketch the graph of $\dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{9} = 1$

Is this not very cool ?

Activity 3 Standard Form

Follow the steps to write the equation in **standard form**:

$$4x^2 - 6y^2 + 16x - 12y - 14 = 0$$

Step 1 Collect the x-terms and y-terms together:

$$= 14$$

Step 2 Factor out the coefficients of x^2 and y^2. **Be careful**: you are factoring a **negative** number from the y^2-terms.

$$= 14$$

Step 3 Complete the square in x and in y.

***** Be careful! ***** Notice that you have really added $4(4) = 16$ to the x-terms and $-6(1) = -6$ to the y-terms on the left side of the equation.

Step 4 Simplify each side.

Step 5 Divide both sides by the constant, and simplify.

Activity 4 Writing the Equation for a Hyperbola

a. Find the equation of the hyperbola shown.

The center of the hyperbola is

$$(h, k) =$$

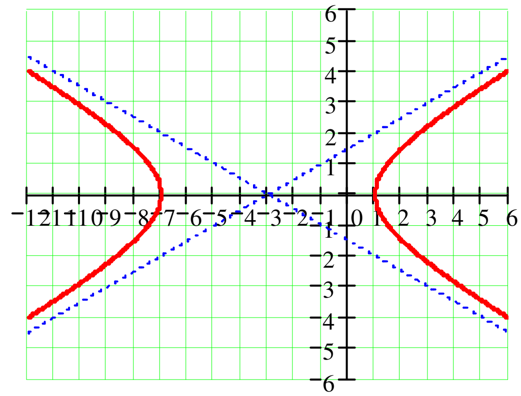
Sketch the central rectangle through the vertices.

The length of the transverse axis is

$2a =$ ________, so $a =$ ________

The length of the conjugate axis is

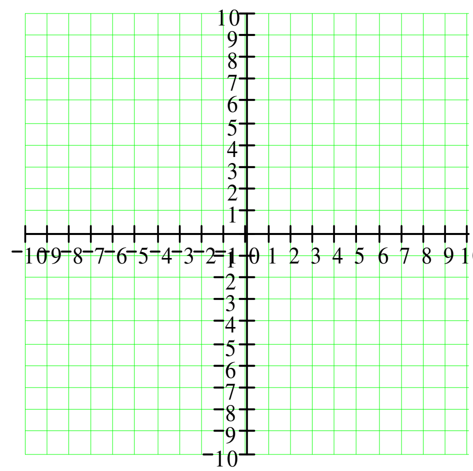
$2b =$ ________, so $b =$ ________



The equation of the hyperbola is:

b. Write the equation of the hyperbola with center at $(6, -2)$, $a = 1$ and $b = 4$, opening up and down.

Sketch the hyperbola on the grid.



Wrap-Up

In this Lesson, we worked on the following skills and goals related to conic sections:

- Write the equation for a hyperbola in standard form
- Graph a hyperbola
- Find the equation for a hyperbola
- Identify a conic section from its equation

Check Your Understanding

1. In Activity 1, how do we sketch the asymptotes of the graph?
2. In Activity 2, what was the difference among the equations of the two hyperbolas and the ellipse?
3. In Activity 3, what were you cautioned to be careful about?
4. In Activity 4, how did you find a and b?

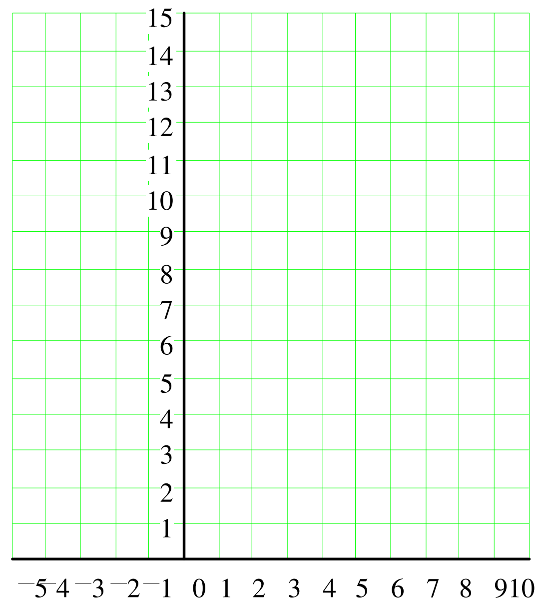
Lesson 43 Inequalities in Two Variables

Activity 1 Solutions of a System of Inequalities

To graph the solution set, graph each inequality *separately*, one at a time. Shade each solution region lightly (or use different colors) so that you can see their intersection.

a. Graph each inequality in the system:

$$5x + 4y < 40$$
$$-3x + 4y < 12$$
$$x < 6, \quad y > 2$$



Inequality 1 Graph $5x + 4y < 40$

x	y
0	
	0

Test Point:

Inequality 2 Graph $-3x + 4y < 12$

x	y
0	
	0

Test Point:

Inequality 3 Graph $x < 6$
(Remember that $x = 6$ is a vertical line. Which side of the line should you shade?)

Inequality 4 Graph $y > 2$
(Remember that $y = 2$ is a horizontal line. Which side of the line should you shade?)

b. Outline the intersection of the four solution sets. Draw dots at the **vertices,** or corners, of the intersection region. (There are four vertices.)

Activity 2 Finding the Vertices

1. Use a system of equations to find the coordinates of each of the vertices of the solution set above.

Intersection of Line 1 and Line 2:

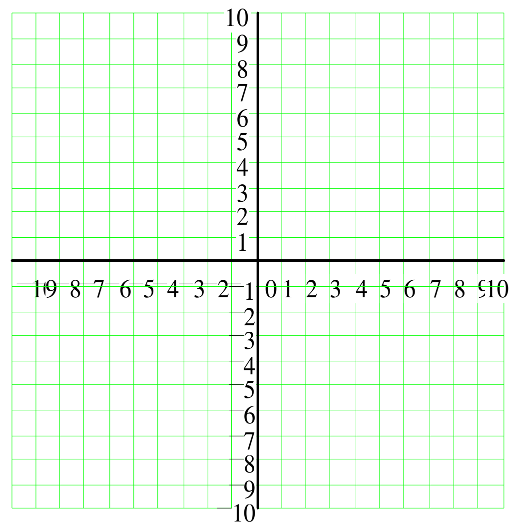
Intersection of Line 1 and Line 3:

Intersection of Line 2 and Line 4:

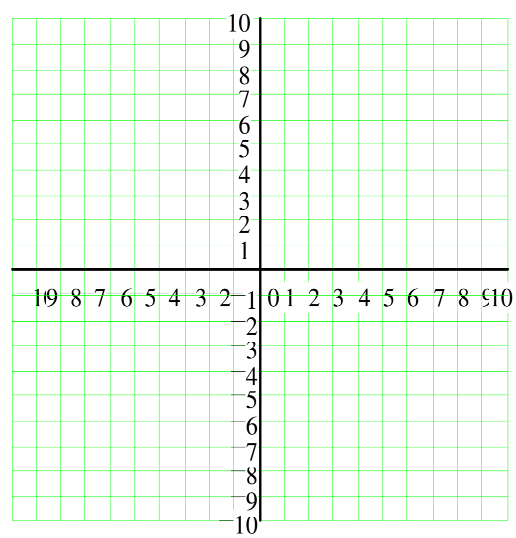
Intersection of Line 3 and Line 4:

2. Graph the solutions to each system of inequalities, and find the vertices of the solution set.
(Note that for part (b), the inequalities $x \geq 0,\ y \geq 0$ restrict the solutions to the first quadrant. This is a common feature of applied problems.)

a. $2y - x < 2$
$x + y \leq 4$



b. $2y + 3x \geq 6$
$2y + x \leq 10$
$y \geq 3x - 9$
$x \geq 0,\ y \geq 0$



Activity 3 Optimization

Grazing animals spend most of their time foraging for food. A small animal such as a ground squirrel must also be alert for predators. Which foraging strategy favors survival: Should the animal look for foods that satisfy its dietary requirements in minimum time, thus minimizing its exposure to predators and the elements, or should it try to maximize its intake of nutrients?

The Columbian ground squirrel eats forb, a type of flowering weed, and grass. One gram of forb provides 2.44 kilocalories, and one gram of grass provides 2.26 kilocalories. For survival, the squirrel needs at least 100 kilocalories per day.

However, the squirrel can digest no more than 314 wet grams of food daily. Each dry gram of forb becomes 2.67 wet grams in the squirrel's stomach, and each dry gram of grass becomes 1.64 wet grams. In addition, the squirrel has at most 342 minutes available for grazing each day. It takes the squirrel 2.05 minutes to eat one gram of forb and 5.21 minutes to eat one gram of grass.

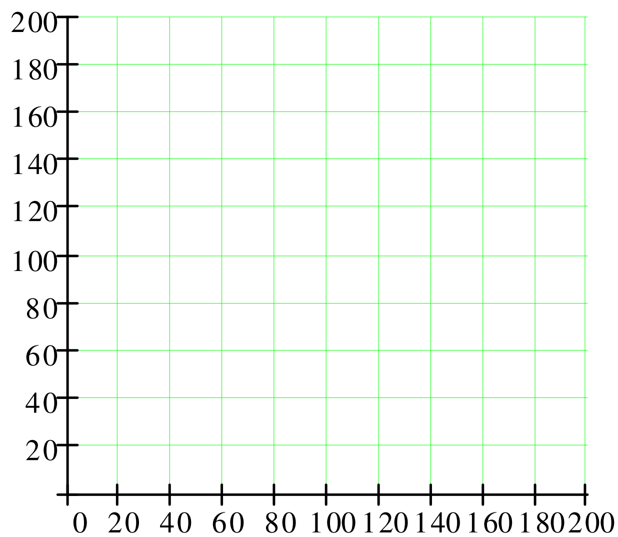
Part I Let x stand for the number of grams of forb the squirrel eats daily, and let y stand for the number of grams of grass.

a. Write an inequality about the amount of energy the squirrel needs.

b. Write another inequality about the amount of food the squirrel can digest.

c. Write a third inequality about the amount of time the squirrel spends eating.

d. Use your calculator to help you graph the inequalities in the first quadrant, and shade the solution set.



Use notebook paper to write your solutions to Parts II and III.

Part II Interpret the solutions:

a. Give two possible combinations of grass and forb that satisfy the squirrel's feeding requirements.

b. If the squirrel can find only grass on a given day, how much grass will it need to eat? If the squirrel can find only forb, how much forb will it need?

c. Find the vertices of the solution set.

Part III The maximum and minimum values of an algebraic expression in two variables always occur at the vertices of a plane region.

a. Write a formula for the squirrel's energy consumption. Evaluate your formula at each of the vertices of the solution set. Round your answers to one decimal place. What is the optimum diet for the squirrel if it wants to maximize its energy consumption?

b. Write a formula for the squirrel's foraging time. Evaluate your formula at each of the vertices of the solution set. Round your answers to one decimal place. What is the optimum diet for the squirrel if it wants to minimize its time foraging?

c. The actual observed diet of the squirrel consists of approximately 100 grams of forb and 25 grams of grass. Does the squirrel appear to be trying to maximize its caloric intake or to minimize its time foraging?

Wrap-Up

In this Lesson, we worked on the following skills and goals related to equations and graphs:

- Graph the solution set for an inequality in two variables
- Graph the solution set for a system of inequalities
- Find the vertices of the solution set

Check Your Understanding

1. What is the difference between the graph of an equation in two variables and an inequality in two variables?

2. When is it wrong to use $(0, 0)$ as a test point? What should you do instead?

3. What does it mean to shade the intersection of two regions?

4. What portion of the Cartesian plane is described by the inequalities $x > 0$ and $y > 0$?

Lesson 44 Nonlinear Systems

Activity 1 Wildlife Management and Sustainable Yield

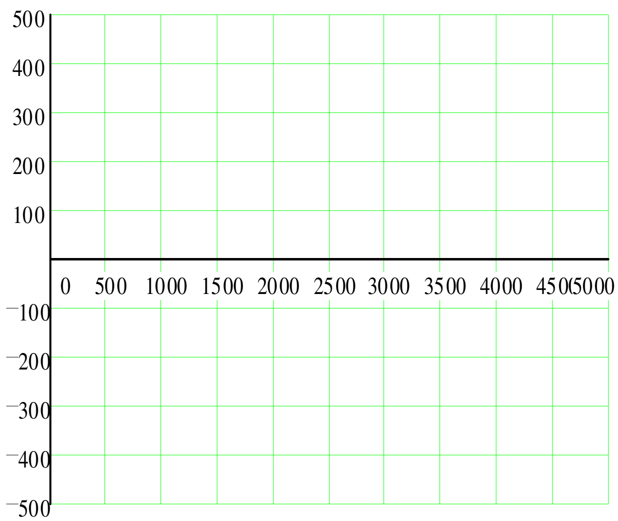
The rate of growth of a population depends on its size. Larger populations grow faster than smaller ones. However, if the population grows too large, its rate of growth will decrease. (Why does this happen?) Many species of fish are threatened by overfishing. It has become important to monitor their population growth, and to understand how harvesting affects the population.

1. The annual growth rate, y, of a population of fish is given by

$$y = 0.4x - 0.0001x^2$$

where x is the current biomass of the population, in tons, and y is in tons per year.

a. Graph y on your calculator, setting Xmax $= 4700$. Then sketch the graph on the grid.



b. Locate the point on the graph with $x = 2500$. What do the coordinates of this point tell you about the fish population?

c. If the biomass is currently 2500 tons, will the population be larger or smaller next year? By how much?

What if the biomass is currently 3500 tons?

d. What size biomass remains stable, i.e. neither grows nor declines?

2. (Continued from Problem 1.)

a. Suppose fishermen harvest 300 tons of fish each year. Sketch the graph of the horizontal line $H = 300$ on the same axes with your graph of y.

b. If the biomass is currently 2500 tons and 300 tons are harvested by fishermen, will the population be larger or smaller next year? By how much?

What if the biomass is currently 3500 tons?

c. What size of biomass will remain stable (neither increase or decrease) from one year to the next if 300 tons of fish are harvested? Why?

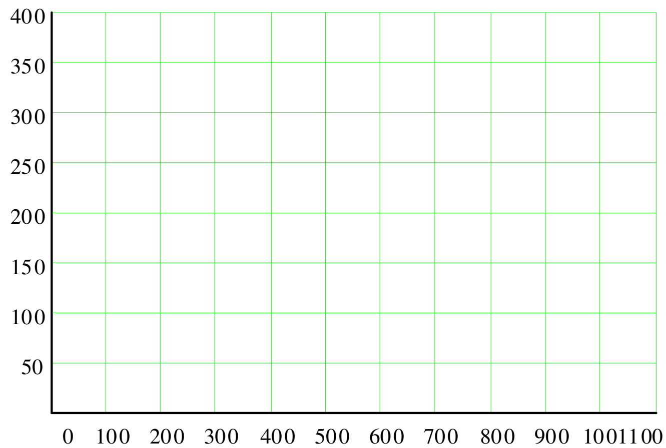
d. If the biomass ever falls below 1000 tons, what will happen after several years of harvesting 300 tons annually?

3. The annual increase in the biomass of a whale population is given in tons by

$$w = 0.001x(1000 - x)$$

where x is the current population in tons, and w is the rate of growth in tons per year.

a. Sketch a graph of w for $0 \le x \le 1100$. What size biomass remains stable?



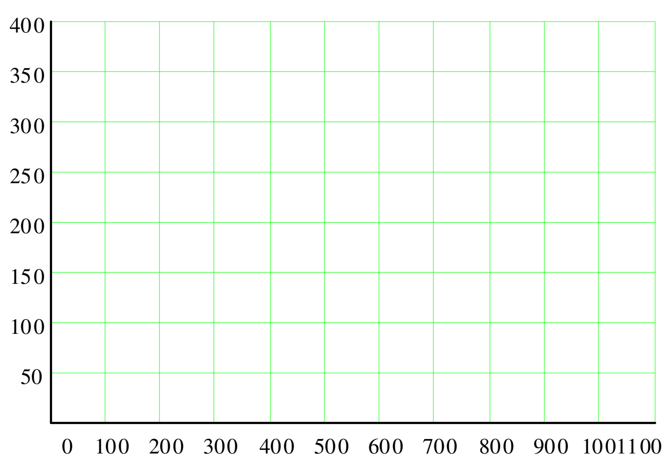
b. Each year hunters are allowed to harvest a biomass given by $H = 0.6x$. Sketch H on the same graph with w. What is the stable biomass with hunting?

c. What sizes of population will increase despite hunting? What sizes will decrease?

d. What size will the population approach over time? What biomass are hunters allowed to harvest for that size population?

4. (Continued from Problem 3.)

a. On the grid below, make another sketch of the graph of $w = 0.001x(1000 - x)$.



b. Find a value of k so that the graph of $H = kx$ will pass through the vertex of $w = 0.001x(1000 - x)$. Sketch $H = kx$ on your graph above.

c. For the value of k found in part (a), what size will the population approach over time?

What biomass are hunters allowed to harvest for that size population?

d. Explain why the whaling industry should prefer hunting quotas of $H = kx$ rather than $H = 0.6x$ for a long-term strategy, even though $0.6x > kx$ for any positive value of x.

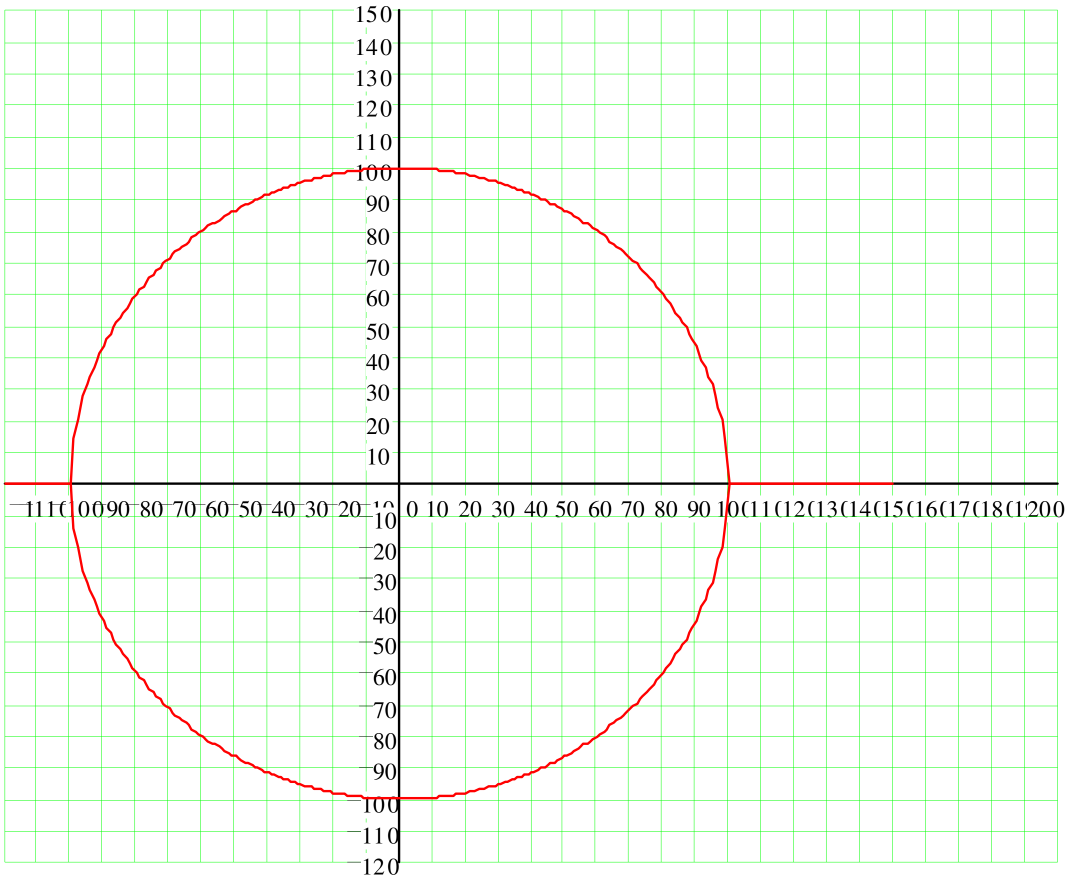
Activity 2 Global Positioning

The Global Positioning System (GPS) uses a collection of satellites in orbit around the earth. Each GPS satellite transmits its position and the current time at regular intervals. A person with a GPS receiver can calculate his or her distance from the satellite by comparing the time of transmission with the time when it receives the signal. Of course, there are many points at the same distance from the satellite—in fact, the set of all points at a fixed distance r from the satellite lie on a sphere centered at the satellite. That is why there are several satellites: You calculate your position by finding the intersection point of several such spheres centered on different satellites.

We will consider a simplified, two-dimensional model of a GPS system in which the satellites and the receiver all lie in the xy-plane, instead of in three-dimensional space. In this model we'll need data from two GPS satellites. The satellites are orbiting along a circle of radius 100 meters centered at the origin and the transmission signals travel at 5 meters per second. You are inside that circle, and would like to know your coordinates.

a. A signal from Satellite A arrives 18 seconds after it was transmitted. How far are you from Satellite A?

b. The signal says that Satellite A was located at $(100, 0)$ at the time of transmission. Use a compass to sketch a graph showing your possible positions relative to Satellite A.



c. Find an equation for the graph you sketched in part b.

d. A signal from Satellite B arrives 8.4 seconds after it was transmitted. How far are you from Satellite B?

e. The signal says that Satellite B was located at $(28, 96)$ at the time of transmission. Use a compass to sketch a graph showing your possible positions relative to Satellite B.

f. Find an equation for the graph you sketched in part e.

g. Your position must lie at the intersection point, P, of your two graphs. Estimate the coordinates of your position from the graph. (Remember that you are within the orbit of the satellites.)

h. Solve a system of equations to find the coordinates of P.

Wrap-Up

In this Lesson, we worked on the following skills and goals related to equations and graphs:

- Solve 2×2 systems involving one or more quadratic equations
- Graph a system and locate the solutions
- Find the vertices of the solution set

Check Your Understanding

1. What does the vertical coordinate measure in Activity 1?
2. In Problem 2c of Activity 1, where does the answer appear on the graph?
3. You drew a circle in part b of Activity 2. What does that circle represent?
4. How well did you answers to parts g and h agree?

Skills and Goals

Here are the Skills and Goals you are responsible for learning in this course.

Skills

Unit 1		**Linear Models**
Lesson 2	1.	Graph a line by the intercept method
	2.	Given a linear equation in standard form, solve for y
Lesson 3	3.	Solve linear equations and inequalities algebraically
	4.	Solve linear equations and inequalities graphically
Lesson 4	5.	Compute slope from a graph
	6.	Recognize a linear model by its constant slope
Lesson 5	7.	Graph a line by the slope-intercept method
	8.	Compute slope using a formula
	9.	Find a linear equation using the point-slope formula
	10.	Find the equation of a line through two points
Unit 2		**Applications of Linear Models**
Lesson 6	11.	Sketch a line of best fit for a scatterplot
Lesson 7	12.	Find the solution to a linear system by graphing
	13.	Identify dependent and inconsistent systems
Lesson 8	14.	Solve a system by substitution
	15.	Solve a system by elimination
Lesson 9	16.	Solve a 3×3 linear system by Gaussian elimination
Unit 3		**Quadratic Models**
Lesson 10	17.	Solve quadratic equations and formulas by extracting roots
	18.	Solve quadratic equations by graphing
Lesson 11	19.	Solve quadratic equations by factoring
	20.	Write a quadratic equation with given solutions
Lesson 12	21.	Graph equations $y = ax^2$, $y = x^2 + c$, and $y = ax^2 + bx$
	22.	Find the intercepts and vertex of these parabolas
Lesson 13	23.	Solve quadratic equations by completing the square
Unit 4		**Applications of Quadratic Models**
Lesson 14	24.	Solve quadratic equations by using the quadratic formula
Lesson 15	25.	Sketch a parabola, showing the vertex and intercepts
	26.	Use the vertex form for a quadratic equation
Lesson 16	27.	Find a quadratic equation through three points
Lesson 17	28.	Solve quadratic inequalities
	29.	Use interval notation

Unit 5	**Functions and Their Graphs**	
Lesson 18	30.	Identify functions
	31.	Use function notation
Lesson 19	32.	Read function values from a graph
Lesson 20	33.	Evaluate cube roots and simplify expressions
	34.	Evaluate absolute value and simplify expressions
	35.	Graph the eight basic functions
Lesson 21	36.	Find a variation equation
Lesson 22	37.	Use absolute value to describe distance
	38.	Solve absolute value equations and inequalities
Unit 6	**Powers and Roots**	
Lesson 23	39.	Simplify expressions with negative exponents
	40.	Solve equations involving negative exponents
	41.	Calculate with scientific notation
Lesson 24	42.	Evaluate roots
	43.	Use exponential notation for roots
	44.	Solve radical equations
Lesson 25	45.	Convert between radical and exponential notation
	46.	Simplify expressions with rational exponents
	47.	Solve equations involving rational exponents
Lesson 26	48.	Use the distance and midpoint formulas
	49.	Sketch circles
Lesson 27	50.	Simplify radicals
	51.	Simplify expressions involving radicals
	52.	Rationalize denominators
Lesson 28	53.	Solve radical equations
Unit 7	**Exponential Functions**	
Lesson 29	54.	Evaluate exponential models
Lesson 30	55.	Graph exponential functions
	56.	Solve exponential equations with the same base
	57.	Identify exponential functions from a table of values
	58.	Find an exponential function from its graph
Lesson 31	59.	Compute logarithms
	60.	Solve exponential equations base 10
Lesson 32	61.	Use the properties of logarithms to simplify expressions
	62.	Solve logarithmic equations
	63.	Solve exponential equations
Lesson 33	64.	Find doubling time and half-life
	65.	Find growth and decay factors
Lesson 34	66.	Plot values and read logarithmic scales

Unit 8 **Polynomials and Rational Functions**

Lesson 35 67. Find specific terms in a polynomial product
68. Factor the sum or difference of two cubes

Lesson 36 69. Reduce algebraic fractions

Lesson 37 70. Perform operations on algebraic fractions

Lesson 38 71. Simplify complex fractions
72. Polynomial division

Lesson 39 73. Solve fractional equations

Unit 9 **Equations and Graphs**

Lesson 40 74. Find the equations and slopes of horizontal and vertical lines
75. Find the slopes of parallel and perpendicular lines

Lesson 41 76. Sketch an ellipse
77. Find the equation for an ellipse

Lesson 42 78. Sketch a hyperbola
79. Find the equation for a hyperbola
80. Identify the graph of a general quadratic equation in two variables

Lesson 43 81. Graph the solutions of a system of linear inequalities
82. Find the vertices of the solution set

Lesson 44 83. Solve a quadratic system

Goals

Unit 1 **Linear Models**

1. Write a linear model in the form $y =$ (starting value) $+$ (rate) $\times t$
2. Write a linear model in standard form, $Ax + By = C$
3. Interpret the intercepts of a linear model
4. Use slope to solve problems finding Δx or Δy
5. Interpret slope as a rate
6. Find a linear model given two points

Unit 2 **Applications of Linear Models**

7. Use a regression line for interpolation and extrapolation
8. Solve word problems by graphing a system
9. Solve word problems by writing a system: interest, mixture, motion, supply and demand

Unit 3 Quadratic Models

10. Solve problems involving volume
11. Use the Pythagorean theorem to solve problems
12. Solve compound interest problems
13. Interpret graphs of quadratic models
14. Model and solve area problems
15. Use the connections between factors of a quadratic trinomial, solutions of a quadratic equation, and the x-intercepts of a parabola

Unit 4 Applications of Quadratic Models

16. Solve problems requiring the quadratic formula
17. Write an equation for a parabola, given either the intercepts or the vertex
18. Write a quadratic model using the vertex formula
19. Find maximum or minimum values
20. Model revenue problems by a quadratic equation

Unit 5 Functions and Their Graphs

21. Interpret function notation
22. Interpret graphs of functions
23. Graph and interpret asymptotes
24. Choose an appropriate graph for a model
25. Solve variation problems
26. Identify variation from a verbal description, a table of values, or a graph

Unit 6 Powers and Roots

27. Evaluate and interpret power functions (integer exponents)
28. Evaluate and interpret power functions (rational exponents)
29. Find equations for circles

Unit 7 Exponential Functions

30. Write models for exponential growth and decay
31. Interpret exponential models
32. Solve problems involving exponential models
33. Use logarithmic scales
34. Work with doubling time and half-life
35. Use the compound interest formula

Unit 8 Polynomials and Rational Functions

36. Write polynomial models
37. Simplify expressions involving distance, rate, and time
38. Solve problems involving algebraic fractions